

Fundamentals of Solid State Physics

Materials and Crystal Structures

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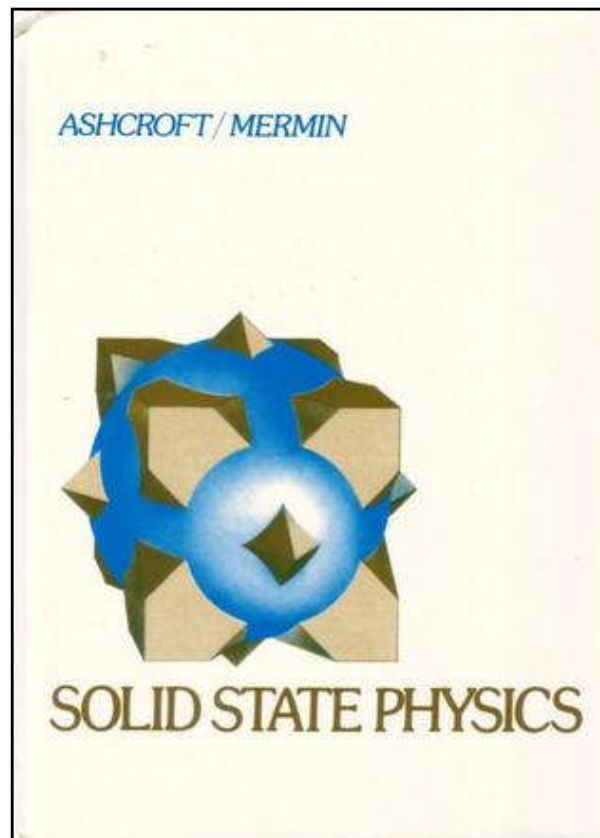
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This Class

- Introduction (Week 1)
- **Materials and Crystal Structures (Week 2–3)**
 - **Crystal structures, lattices**
 - **Reciprocal space, Brillouin zones**
 - **Materials Characterization: Wave diffraction, the Bragg law**
- Electronic Properties (Week 4–12)
- Thermal Properties (Week 13)
- Optical Properties (Week 14)
- Magnetic Properties (Week 15)

Further Reading

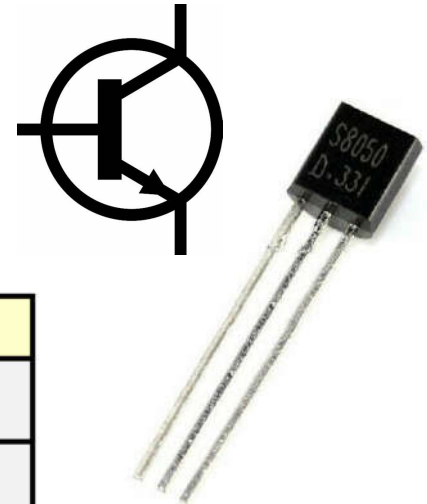
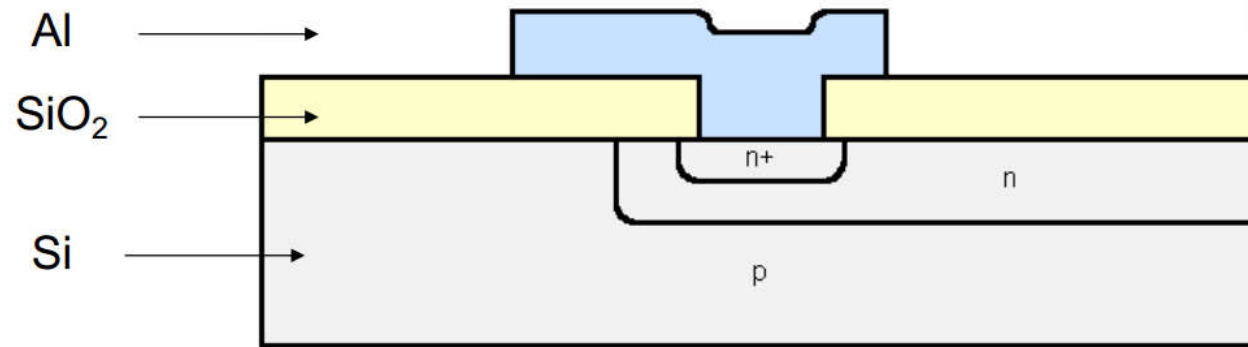
- **Ashcroft & Mermin, Chapter 4, 5, 6**



Importance of Materials

CMOS transistor

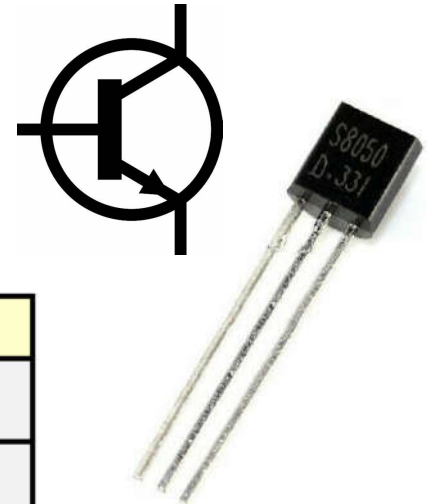
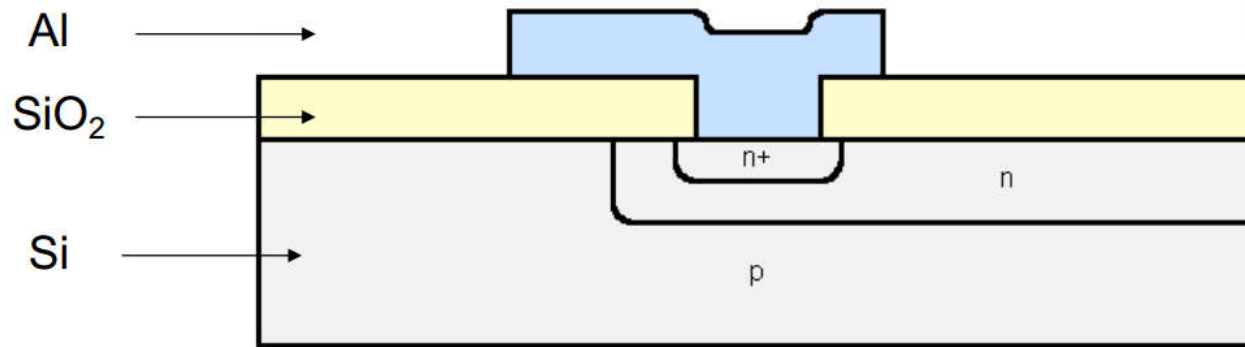
- Complementary **Metal-Oxide-Semiconductor**



Importance of Materials

CMOS transistor

- Complementary **Metal-Oxide-Semiconductor**



Metal



SiO₂



Silicon

Importance of Materials



Metal



SiO₂



Silicon

- **Crystal Structures**
 - polycrystalline, amorphous, single crystalline
- **Electronics**
 - conductor, insulator, semiconductor
- **Optics (in the visible range)**
 - reflective, transparent, absorbing

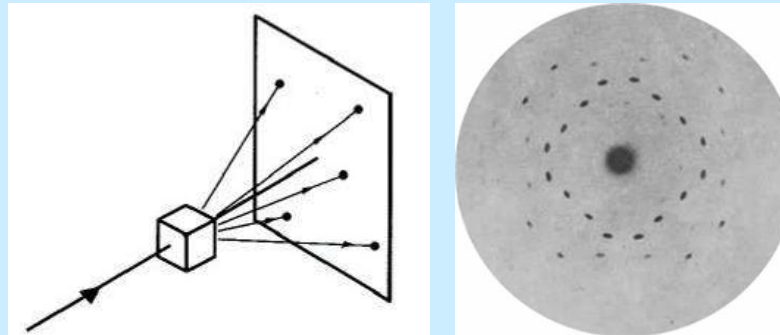
History of Crystal Structures

**Discovery
of X-ray**



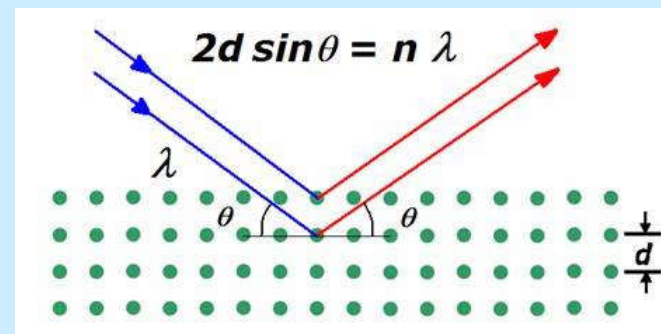
**W. Rontgen (伦琴)
Nobel Prize in 1901**

**X-ray
diffraction
of crystals**



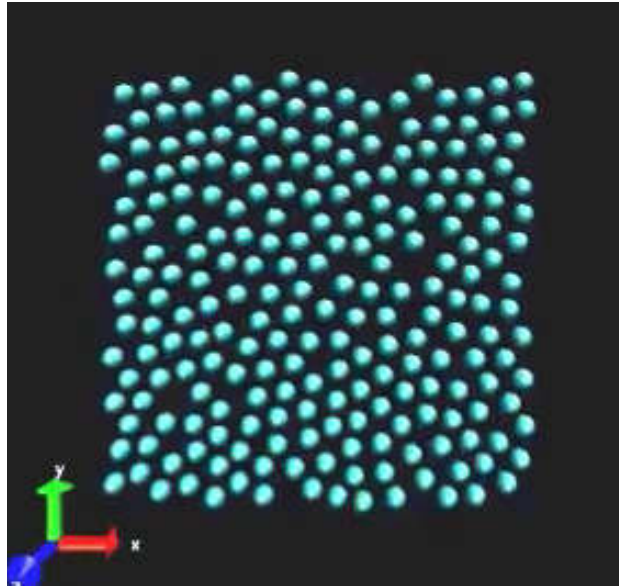
**M. von Laue (劳厄)
Nobel Prize in 1914**

Bragg's law

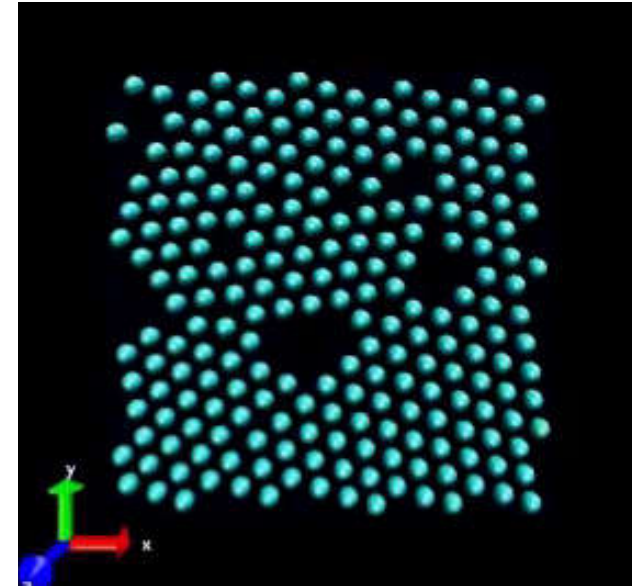


**Bragg & Bragg
(布拉格父子)
Nobel Prize in 1915**

Crystal Structures



→
Video

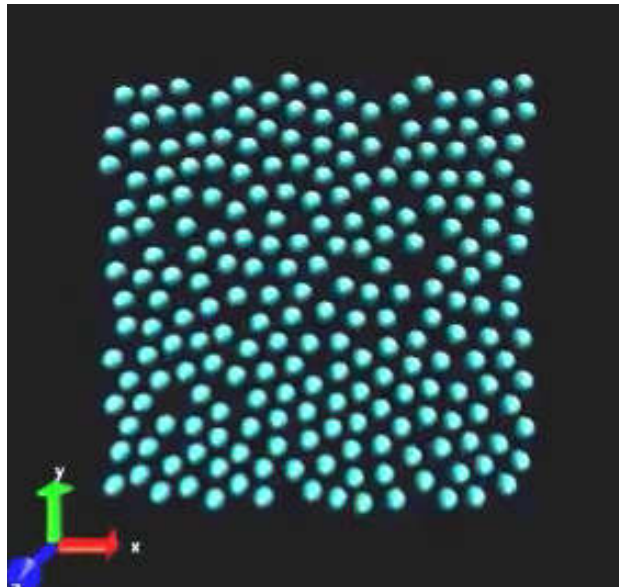


molecular dynamics simulation

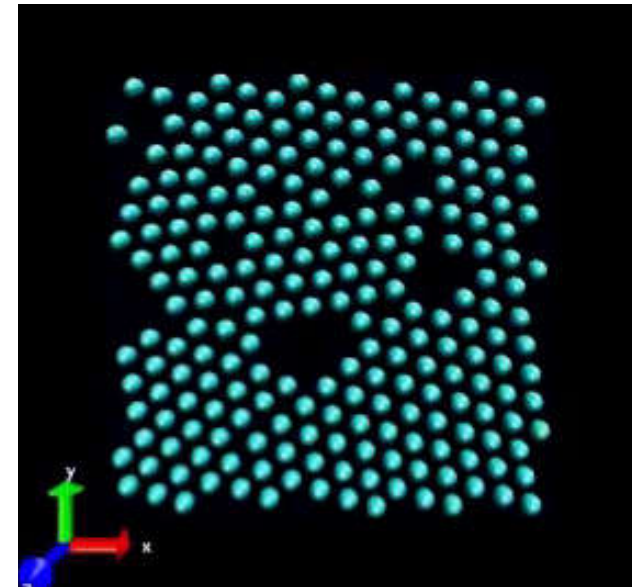
crystal structure:
*ordered, periodic arrays of atoms
(with translational symmetry)*

***Crystal is a microscopic (微观) concept,
not a macroscopic (宏观) concept.***

Crystal Structures



→
Video



our macroscopic world

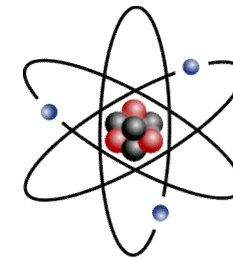


?

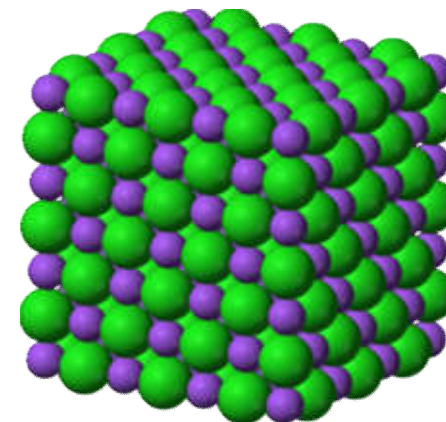
Chemical Bonding 化学键

■ Solids are formed by chemical bonding between atoms

- Metallic Bonding 金属键
- Ionic Bonding 离子键
- Covalent Bonding 共价键
- Van der Waals Bonding 范德华键
- Hydrogen Bonding 氢键
- ...



atom

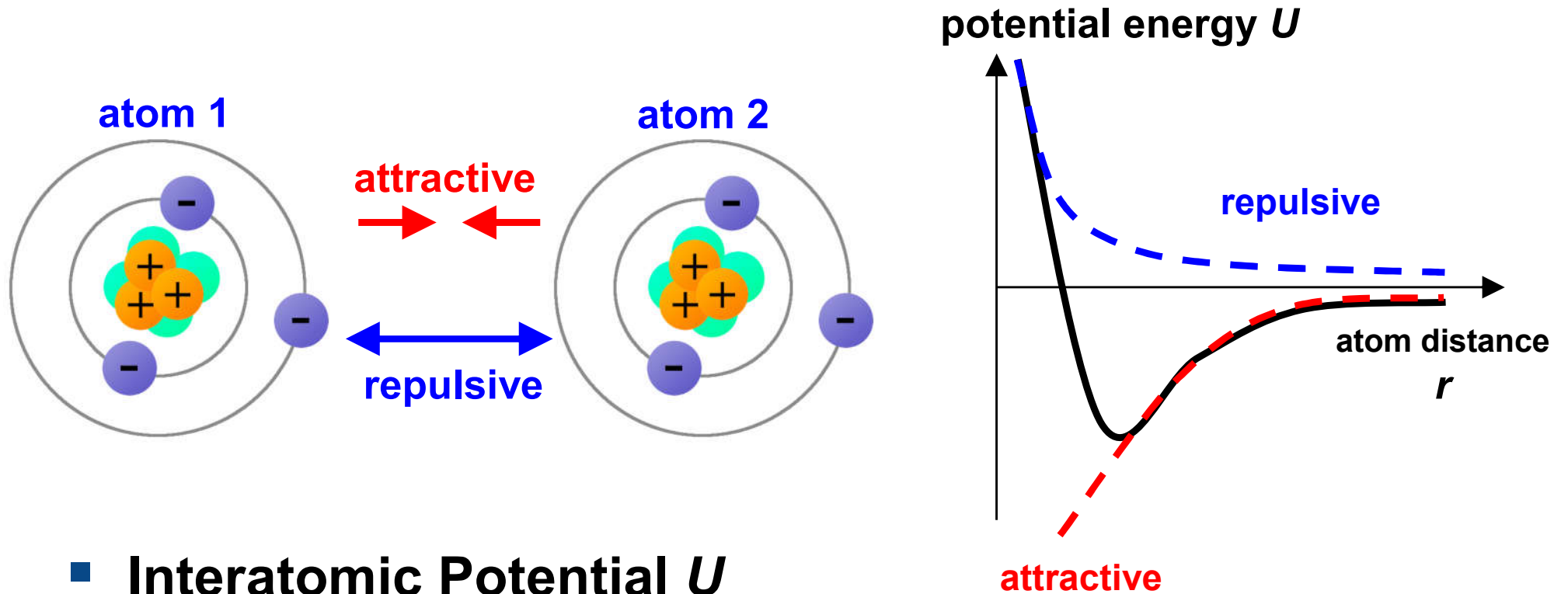


solid

■ Valence electrons form bonds

- Silicon (Si) $[1s^2 2s^2 2p^6] \underline{3s^2 3p^2}$

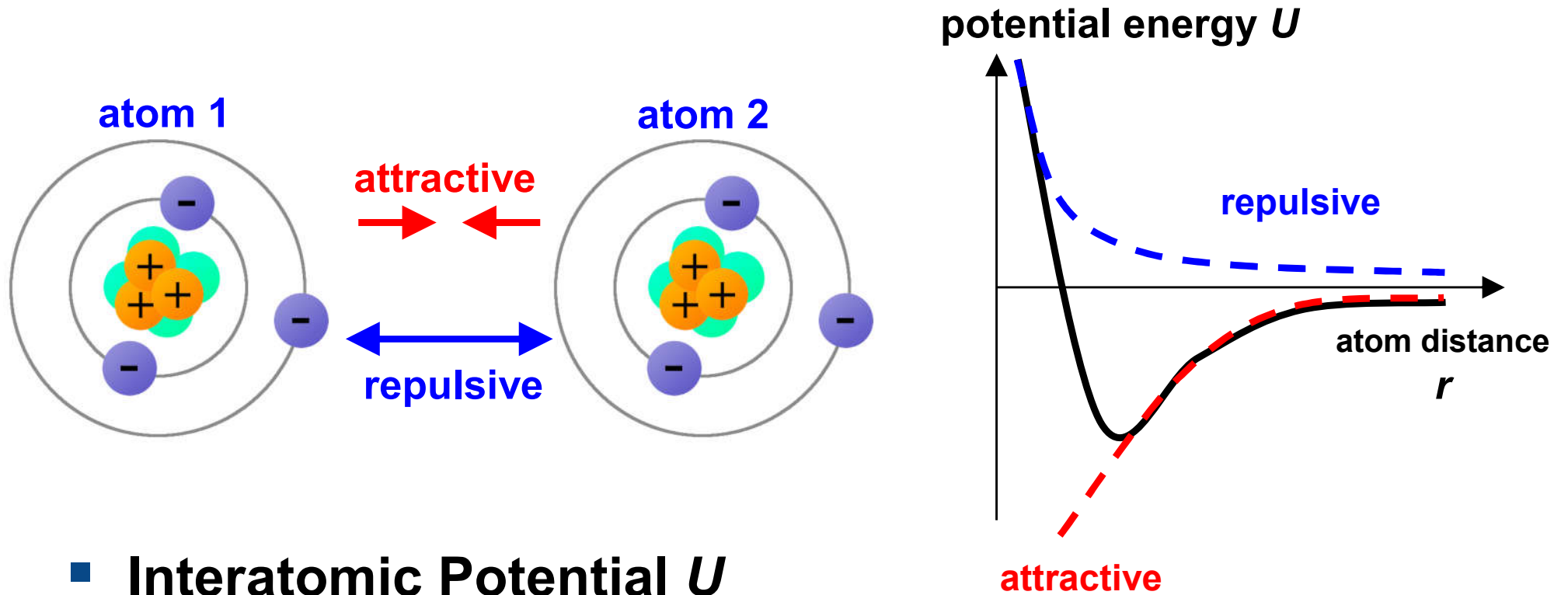
Atomic Interactions



■ Interatomic Potential U

- attraction: electrostatic (+ -)
- repulsion: electrostatic (+ + / - -)
and Pauli exclusion principle

Atomic Interactions

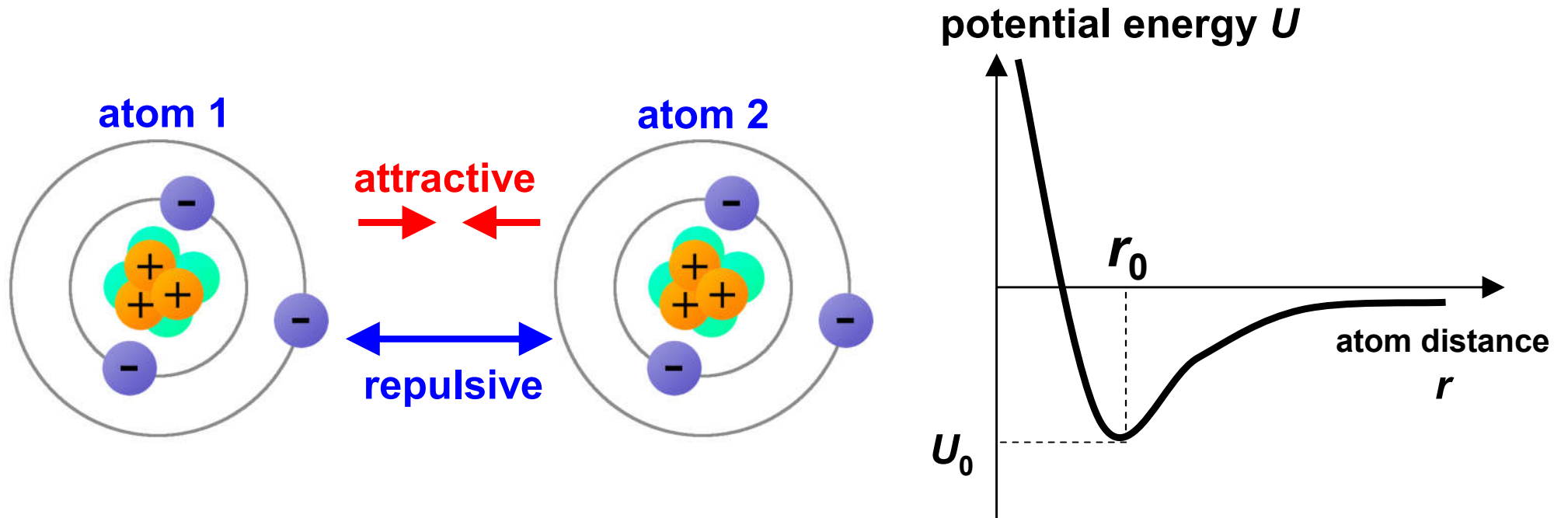


$$U(r) = U_{\text{repulsion}}(r) - U_{\text{attraction}}(r)$$

U - potential energy (J, eV)

r - atomic distance (nm, Å)

Atomic Interactions



■ Interatomic Potential U

$-U_0$ - cohesive energy (结合能)
lowest energy state

$$U(r) = U_{\text{repulsion}}(r) - U_{\text{attraction}}(r)$$

U - potential energy (J, eV)

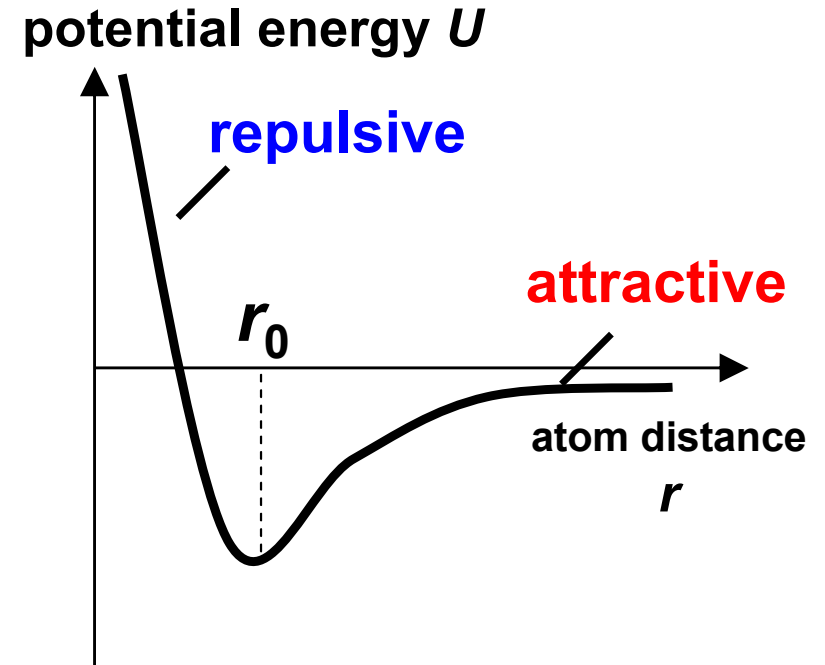
r - atomic distance (nm, Å)

Interatomic Potential: Examples

- Lennard-Jones (L-J)

$$U(r) = \frac{A}{r^{12}} - \frac{B}{r^6}$$

repulsive attractive



- Buckingham Potential

$$U(r) = A \exp\left(-\frac{r}{\rho}\right) - \frac{B}{r^6}$$

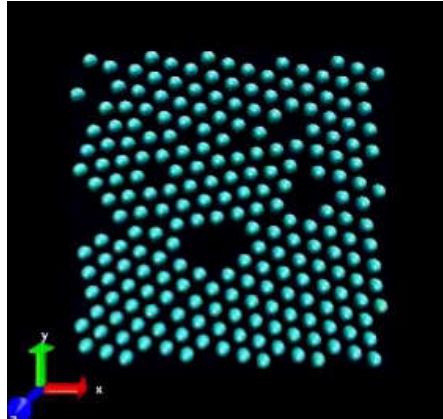
repulsive attractive

- Morse Potential

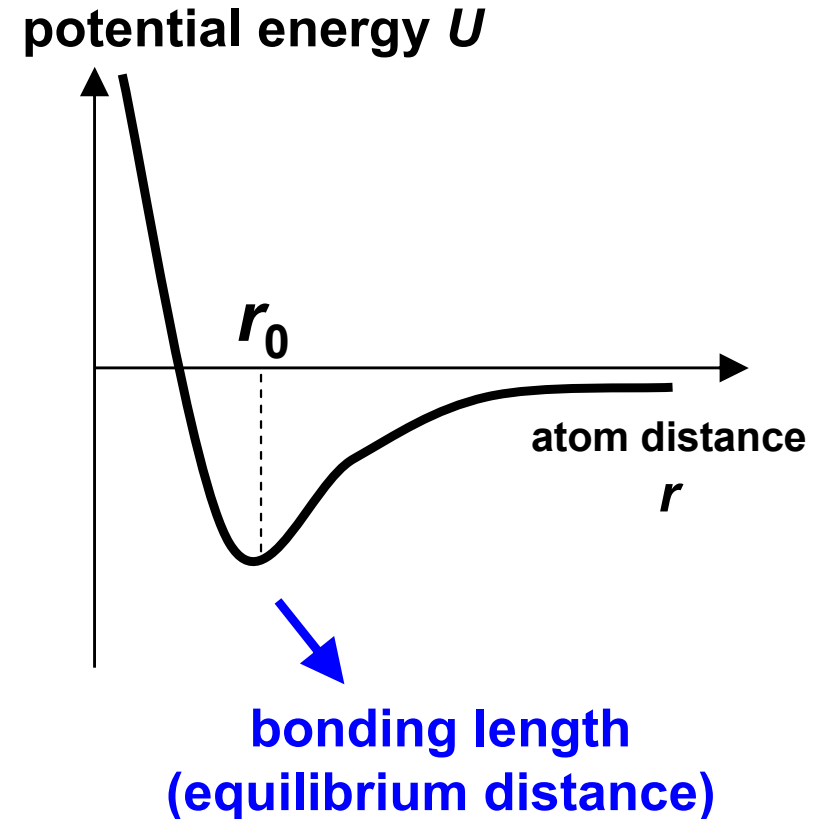
$$U(r) = D \left(e^{-2a(r-r_0)} - 2e^{-a(r-r_0)} \right)$$

repulsive attractive

Atomic Interactions



optimal distance r_0 :
periodicity of crystals



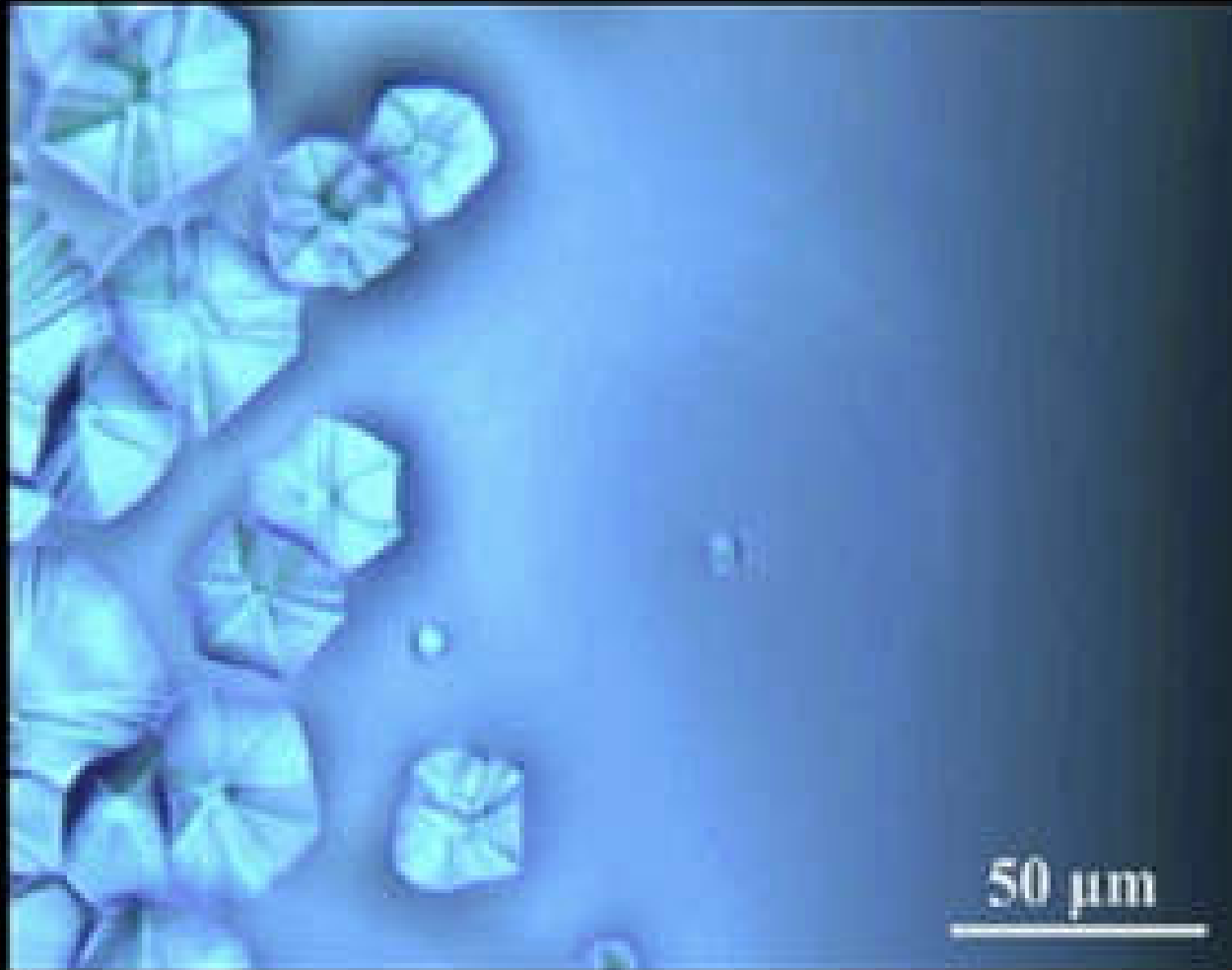
■ Interatomic Potential U

$$U(r) = U_{\text{repulsion}}(r) - U_{\text{attaction}}(r)$$

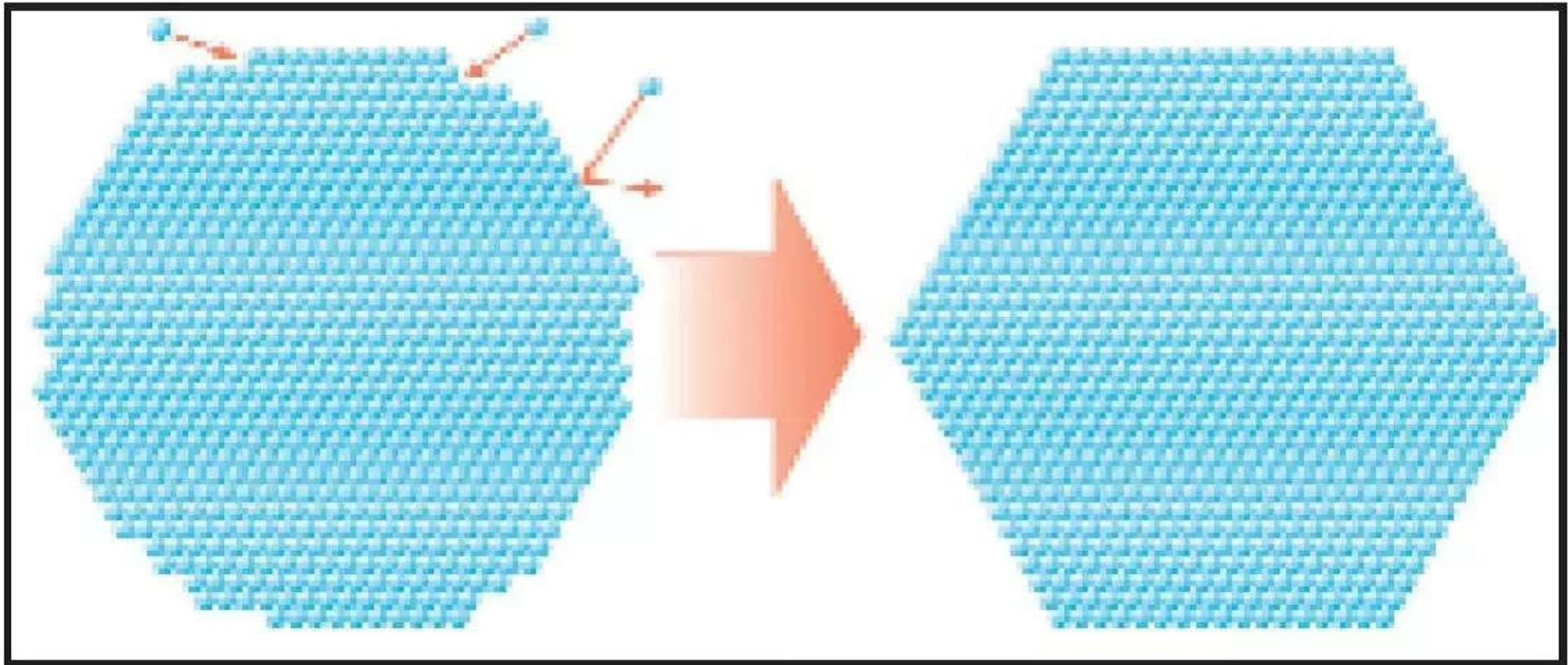
U - potential energy (J, eV)

r - atomic distance (nm, Å)

Macroscopic Crystals



Macroscopic Crystals



Macroscopic Crystals



金刚石 diamond (C)



石英 quartz (SiO_2)



红宝石 ruby ($\text{Al}_2\text{O}_3:\text{Cr}$)

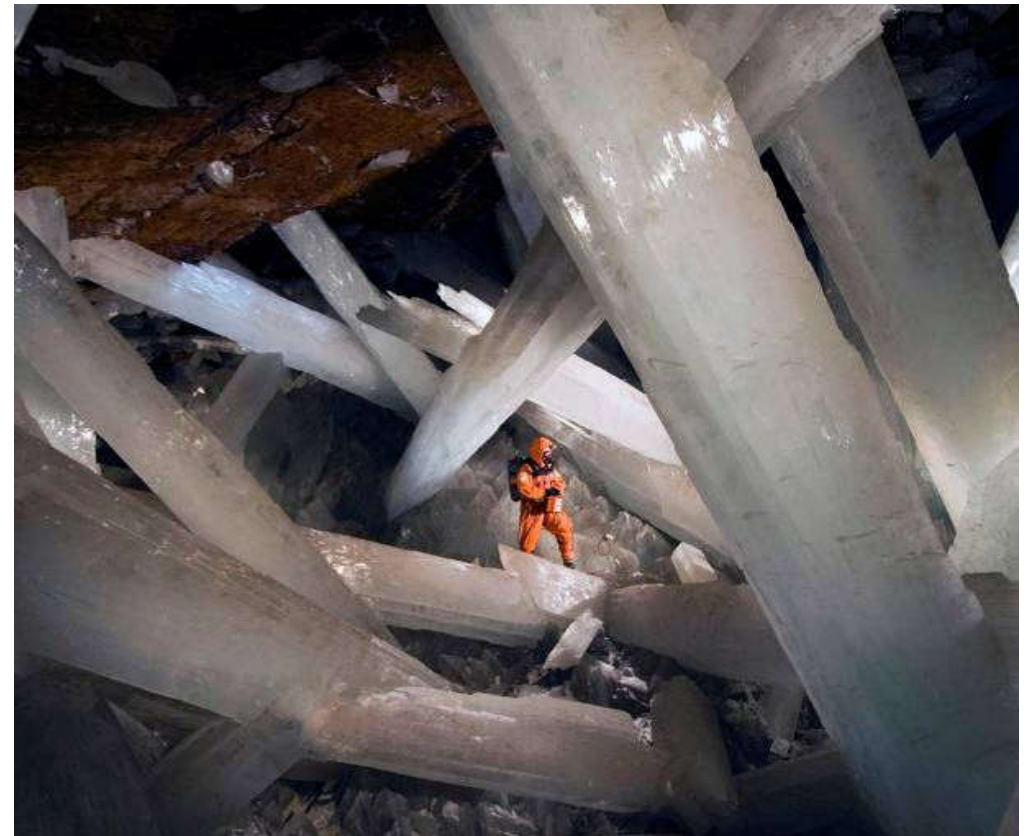


食盐 salt (NaCl)

Macroscopic Crystals



Silicon Crystal



石英 quartz (SiO₂)

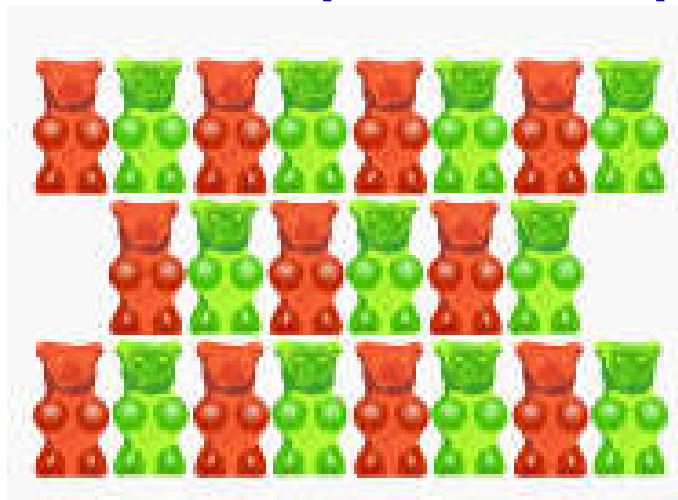
Crystal Cave, Mexico

Crystal Structures

- **Vectors**
 - **Translational vectors**
 - **Primitive vectors**
- **Cells**
 - **Unit Cell, Conventional Cell**
 - **Primitive Cell, Wigner-Seitz Cell**
- **Lattice**
 - **Bravais Lattice**
 - **SC, BCC, FCC, HCP, ...**
- **Packing**
 - **Atomic Packing Factor**
- **Miller Index**
- **Coordination Number**

Crystal, Basis and Lattice

- **Crystal 晶体**
 - real material structures - **physical concept, finite**
- **Basis 基元**
 - single unit of an group of atoms
- **Lattice 晶格/点阵**
 - arrangement of points - **mathematical concept, infinite**
 - one point can represent *one or more* atoms



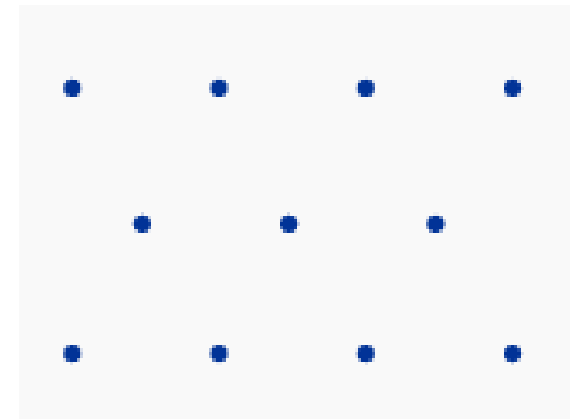
Crystal

=



Basis

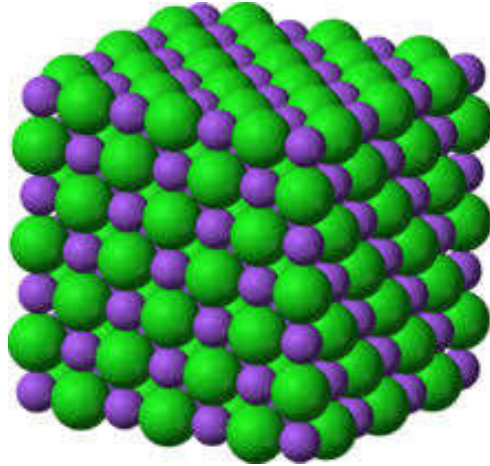
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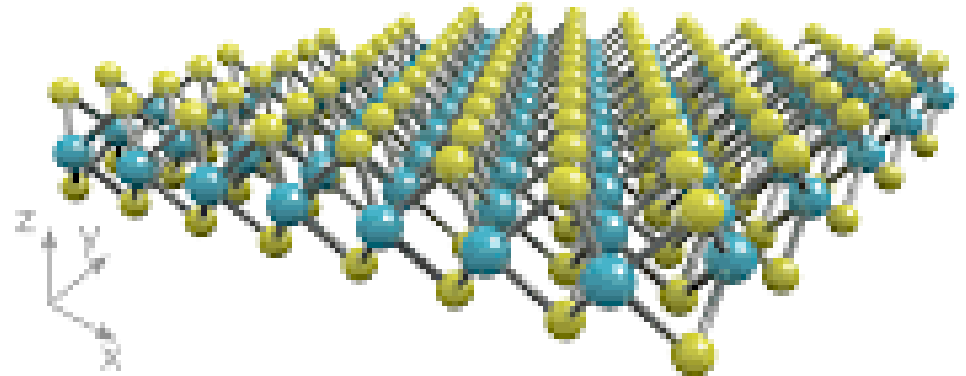
Lattice

Crystal, Basis and Lattice

Crystal

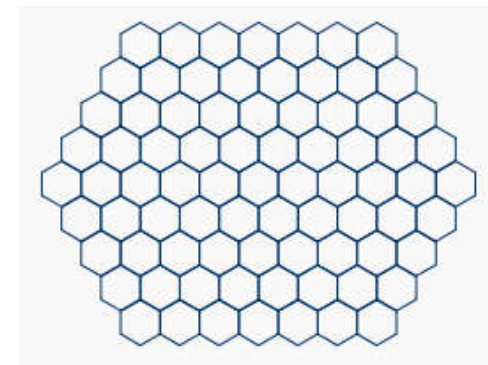
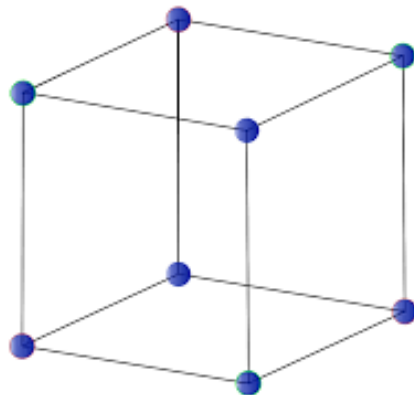


NaCl



monolayer MoS₂

Lattice



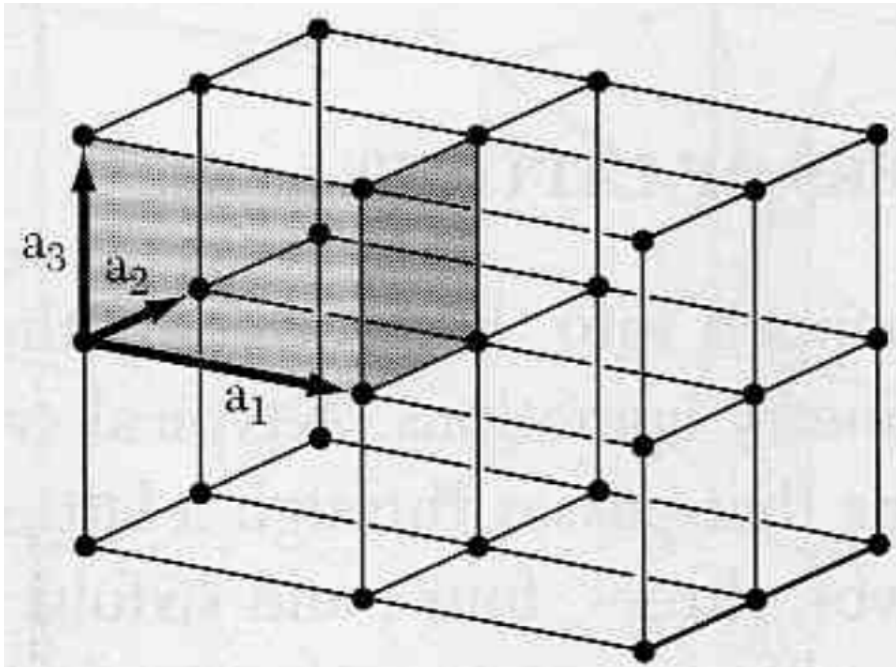
Bravais Lattice 布拉菲点阵

- Each point is *exactly* the same
- Position of each point

$$\mathbf{R} = n_1 \mathbf{a}_1 + n_2 \mathbf{a}_2 + n_3 \mathbf{a}_3$$

n_1, n_2, n_3 cover
all the integers

- $(\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3)$ primitive vectors 基矢量



translational symmetry
平移对称性

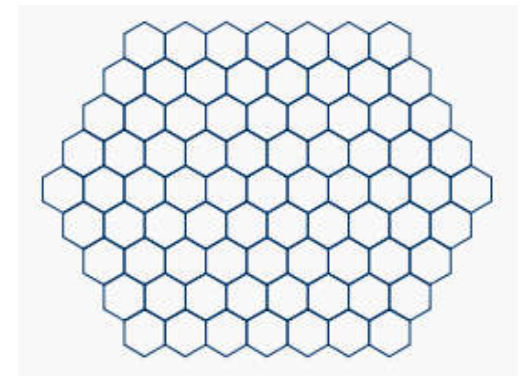
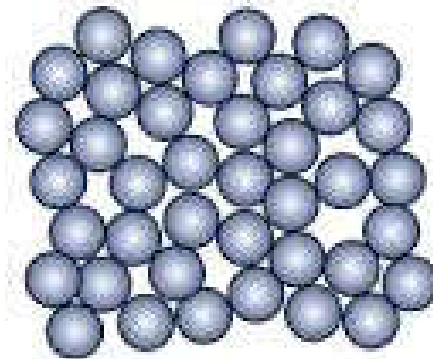
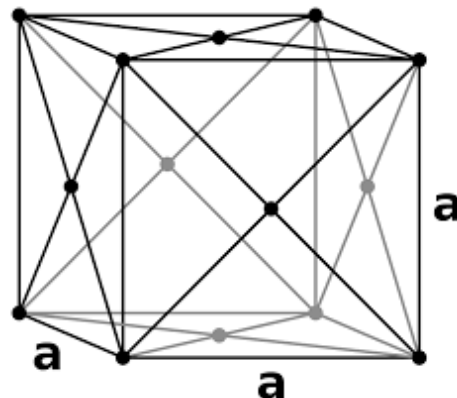
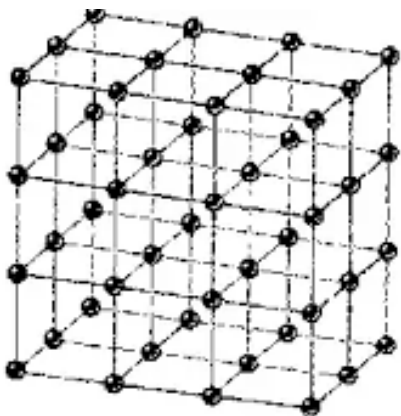
Bravais Lattice 布拉菲点阵

- Each point is *exactly* the same
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$$\mathbf{R} = n_1 \mathbf{a}_1 + n_2 \mathbf{a}_2 + n_3 \mathbf{a}_3$$

n_1, n_2, n_3 cover *all* the integers

Q: which is Bravais lattice, which is not?



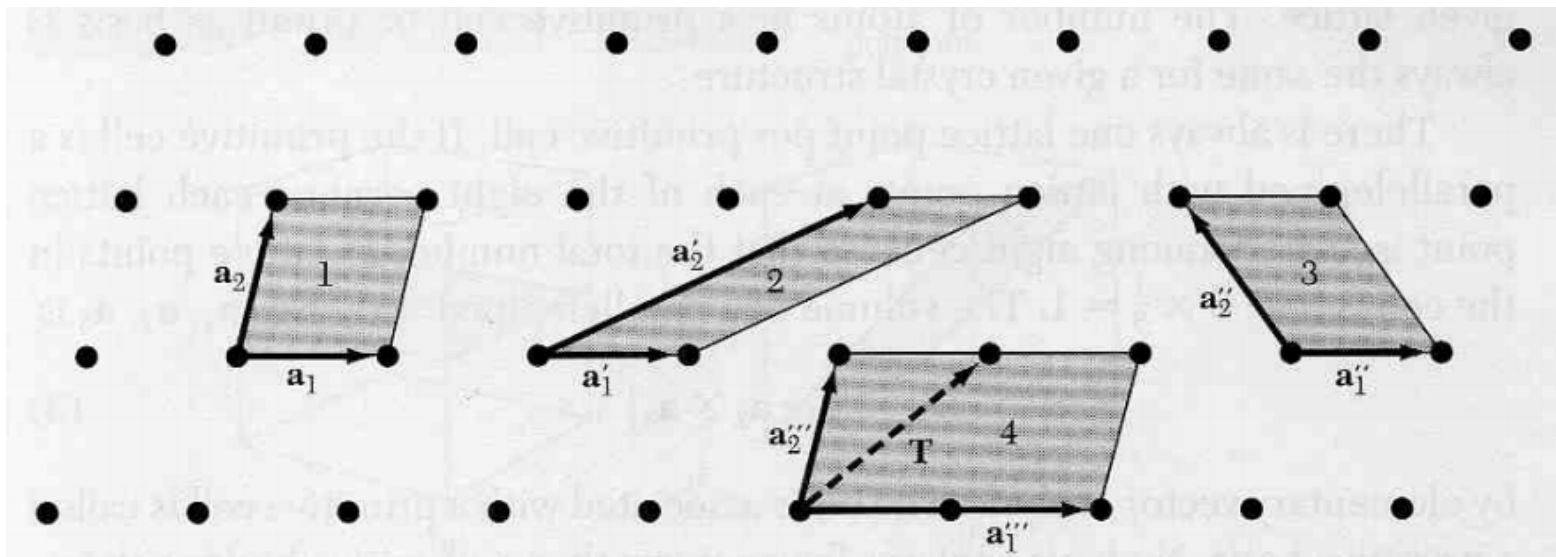
Bravais Lattice 布拉菲点阵

- Each point is *exactly* the same
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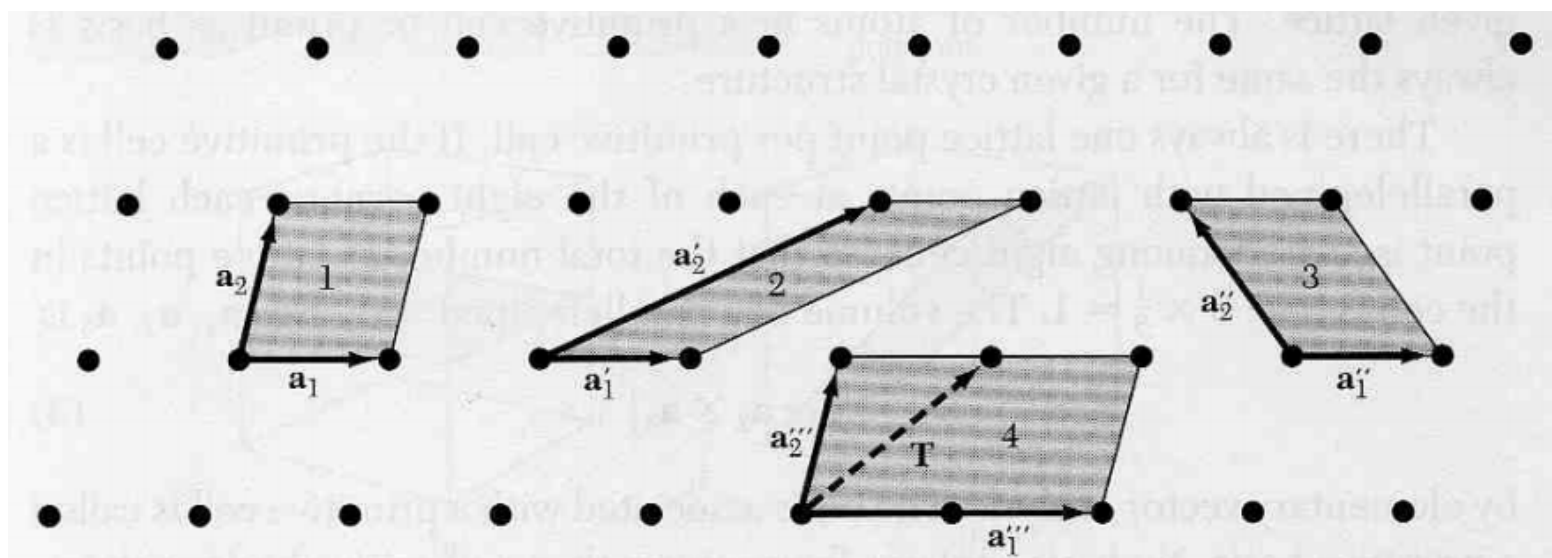
- $(\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3)$ primitive vectors 基矢量



Q: which are primitive vectors, which are not?

Lattice Cells 晶胞

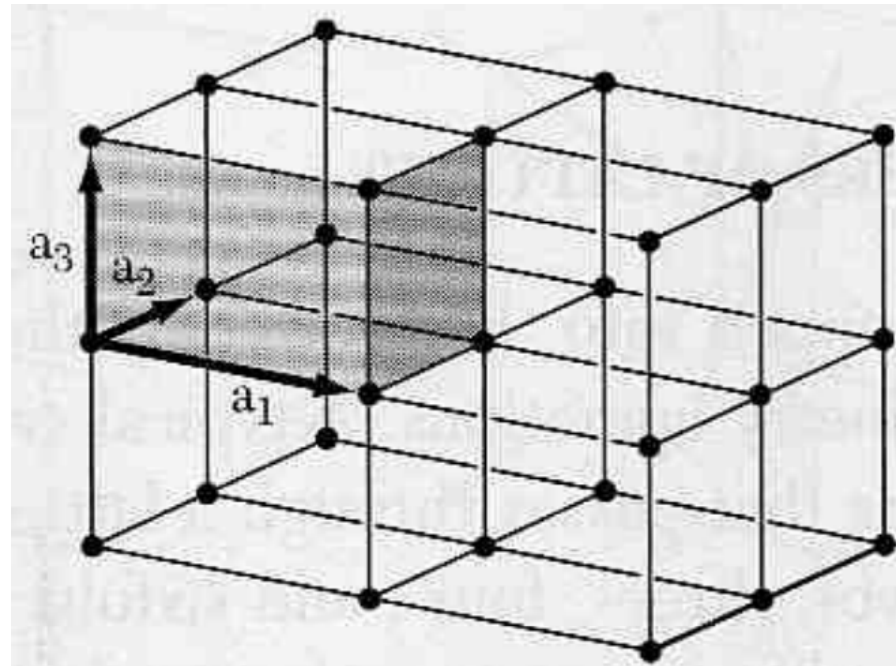
- Repetitive Units to form the infinite lattice



Lattice Cells 晶胞

- Primitive Cell 原胞/素胞
 - A cell with the *smallest* volume
 - A cell with *only one* lattice point

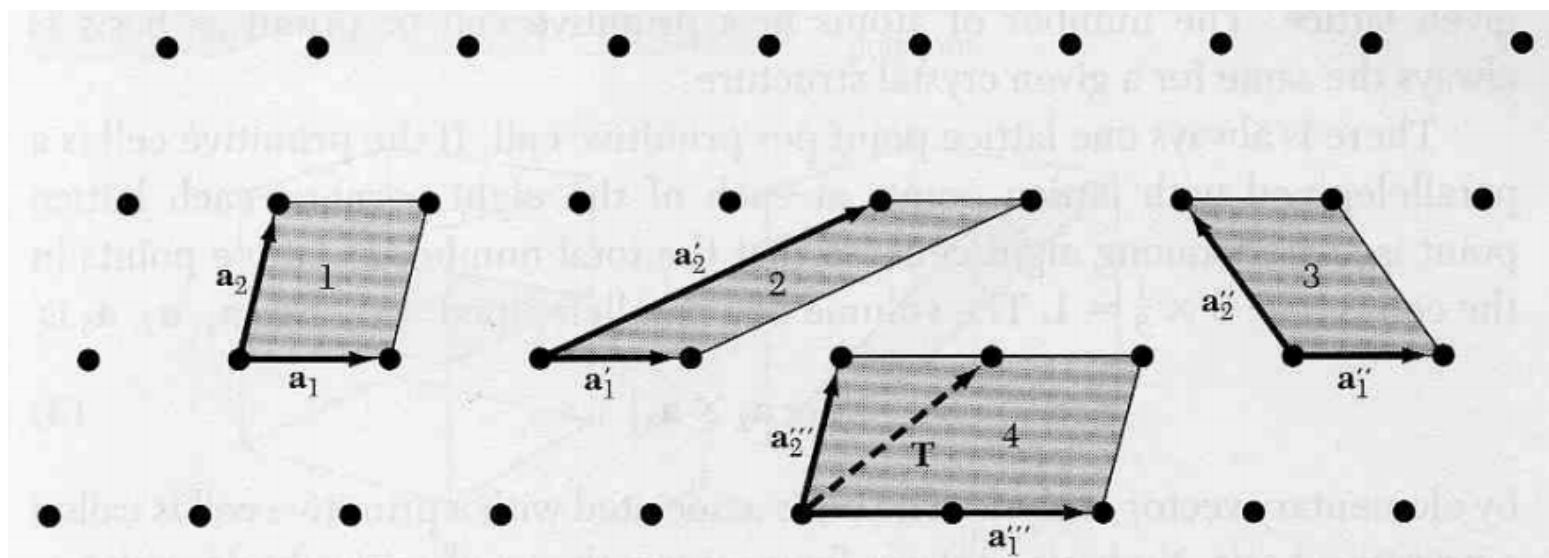
$$V_R = \mathbf{a}_1 \cdot (\mathbf{a}_2 \times \mathbf{a}_3)$$



Q: how many atoms are in this primitive cell?

Lattice Cells 晶胞

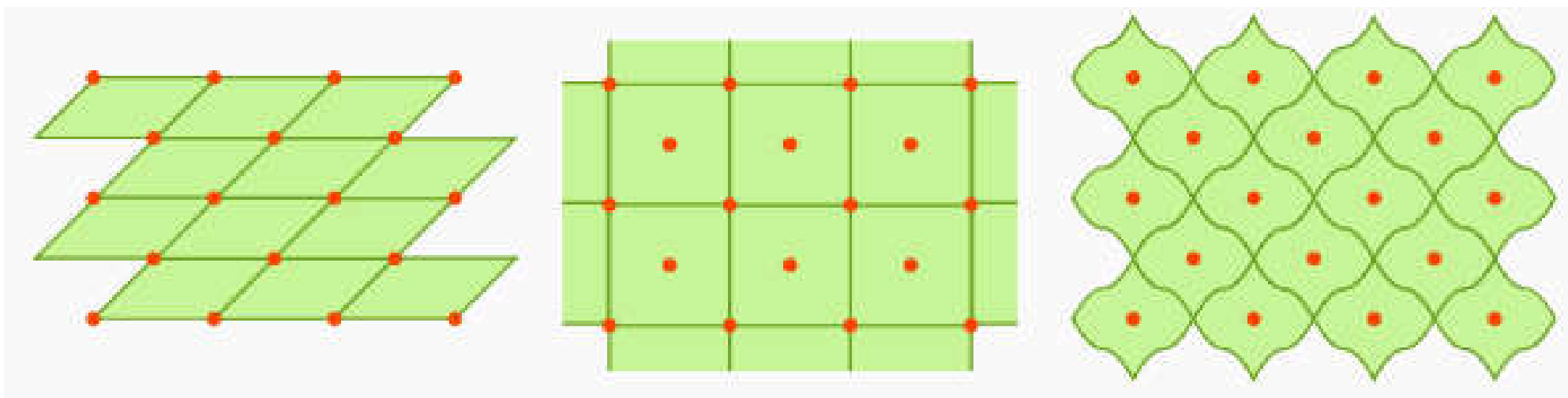
- Primitive Cell 原胞/素胞
 - A cell with the *smallest* volume
 - A cell with *only one* lattice point



Q: which are primitive cells, which are not?

Lattice Cells 晶胞

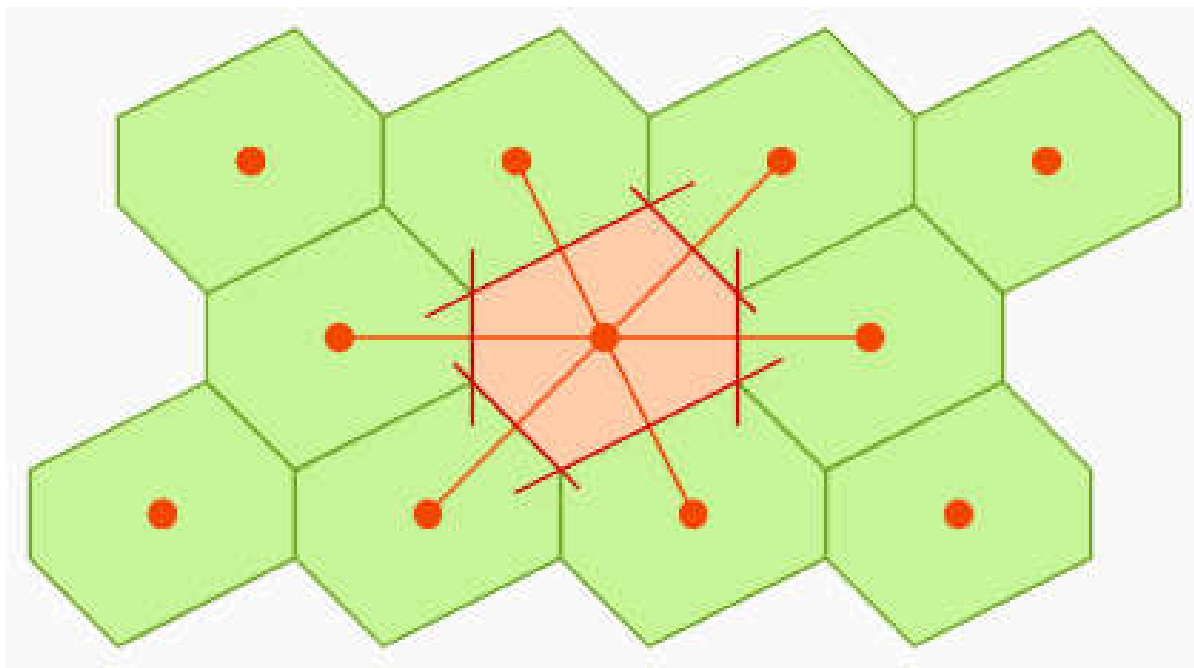
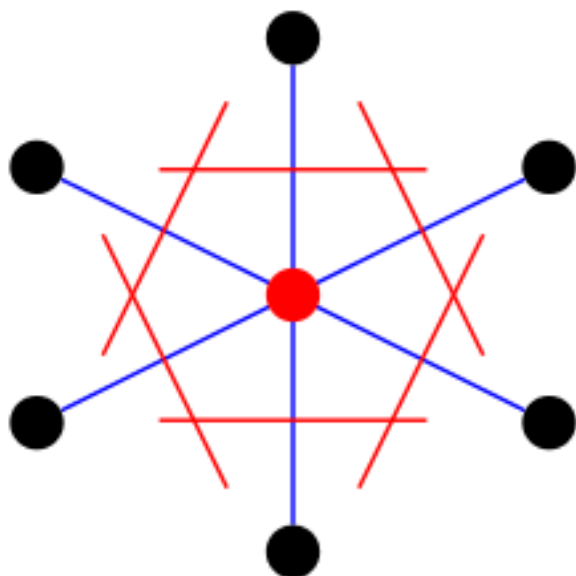
- Primitive Cell 原胞/素胞
 - A cell with the *smallest* volume
 - A cell with *only one* lattice point



Q: which are primitive cells, which are not?

Lattice Cells 晶胞

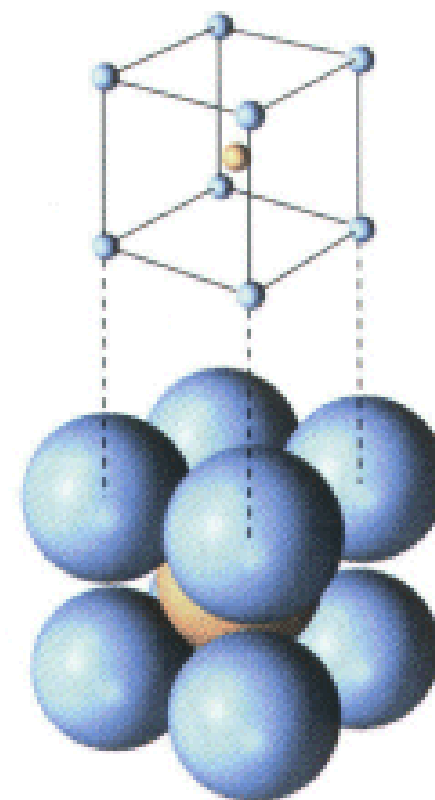
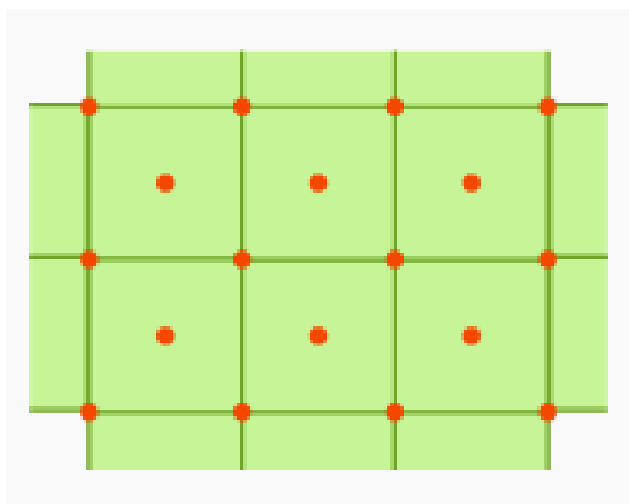
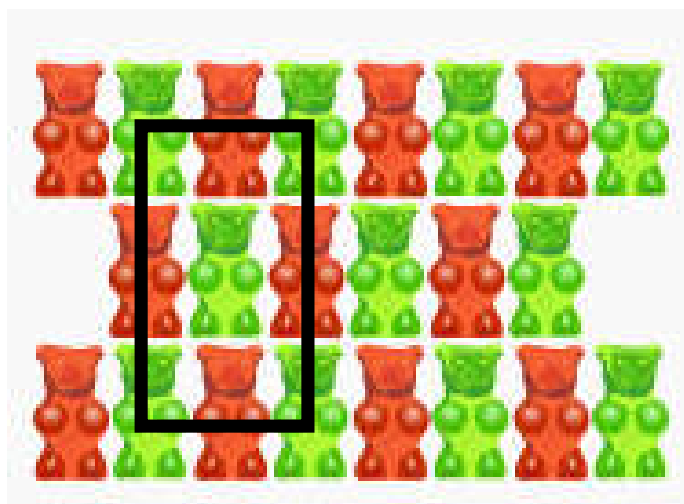
- Wigner-Seitz cell
 - A very special primitive cell



- 1. draw lines to connect nearby points*
- 2. at the midpoint and normal to these lines draw new lines/planes*

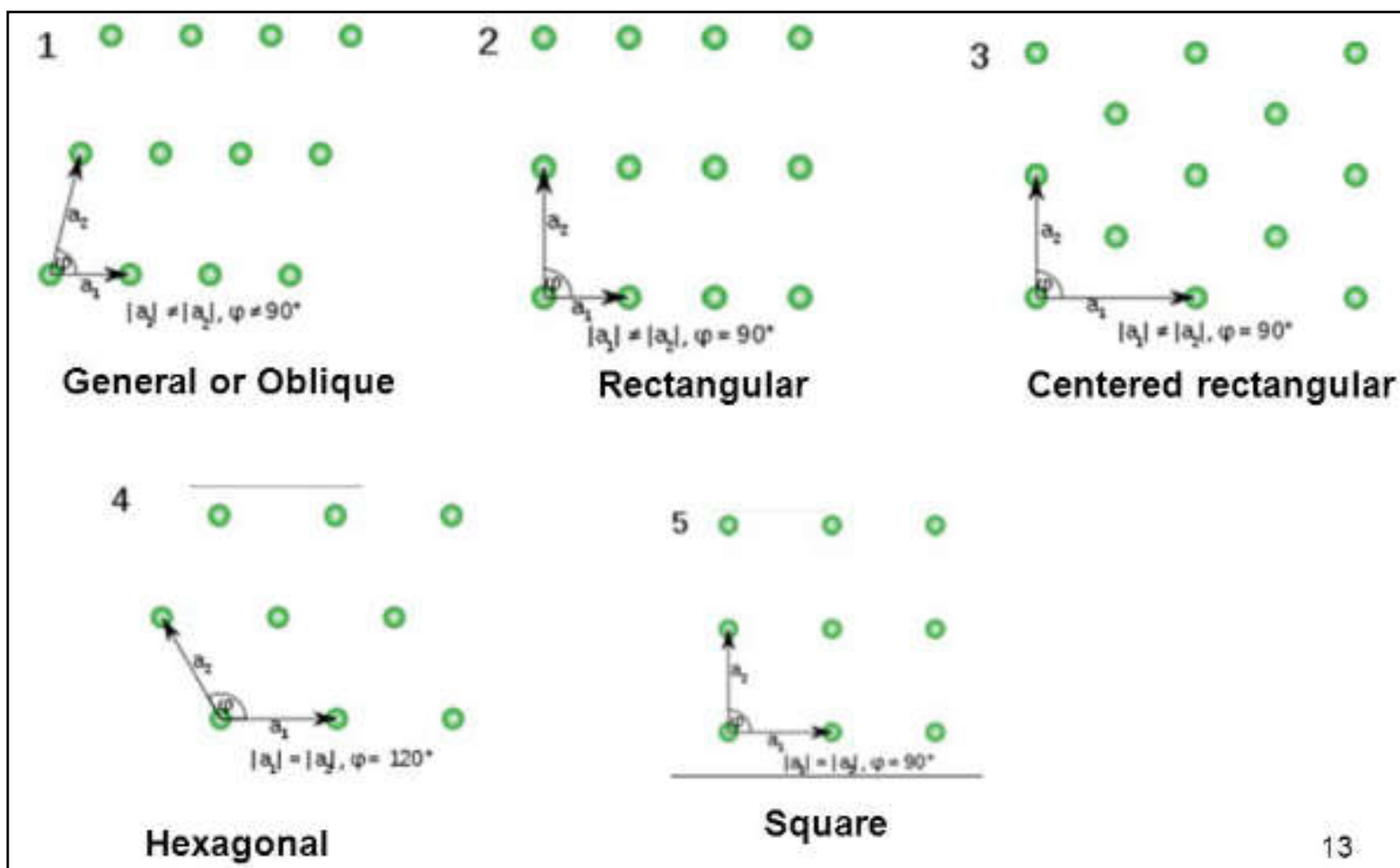
Lattice Cells 晶胞

- Compound Cell 复胞
 - A cell with *more than one* lattice point



2D Bravais Lattice

- There are 5 Bravais lattices in 2D



3D Bravais Lattice

- There are 14 Bravais lattices in 3D

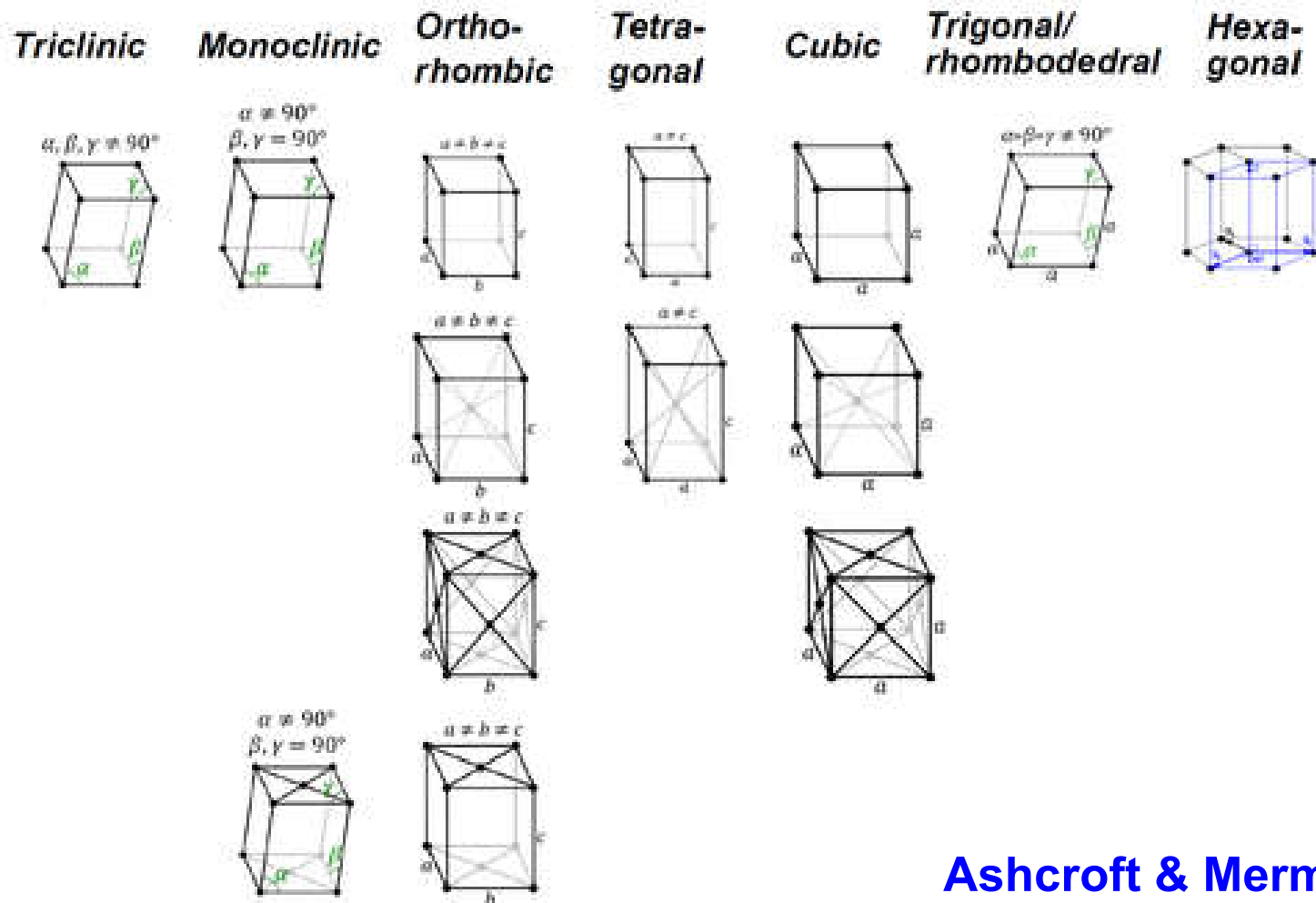
7 crystal systems
(晶系)

P
Simple/Primitive

I
Body Centered

F
Face Centered

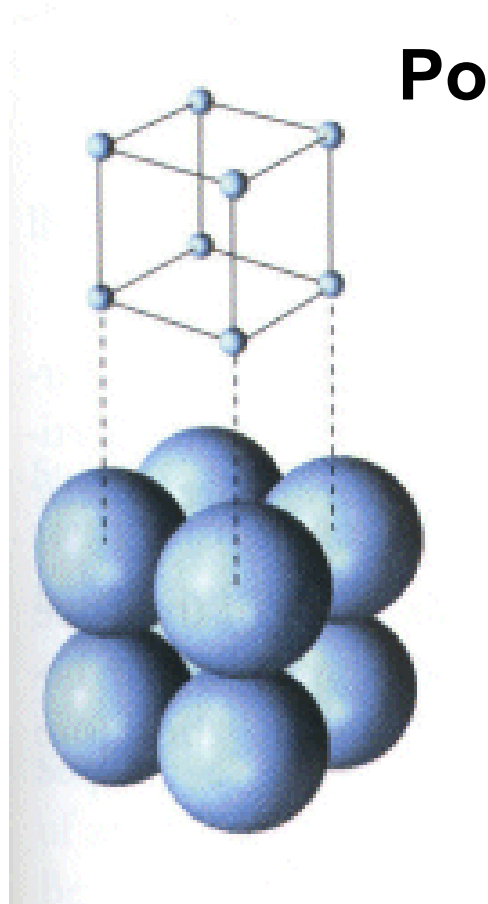
C
Base Centered



Bravais Lattice for Cubics 立方晶格

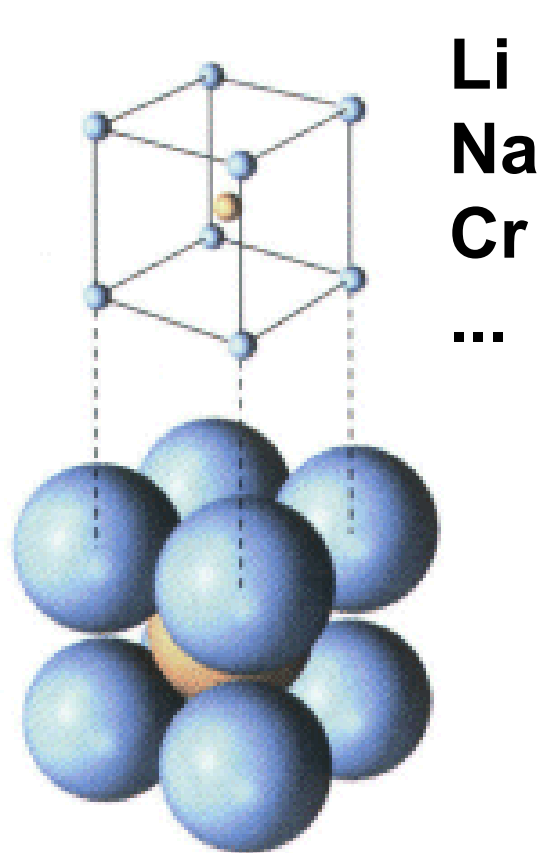
简单立方 SC

Simple cubic



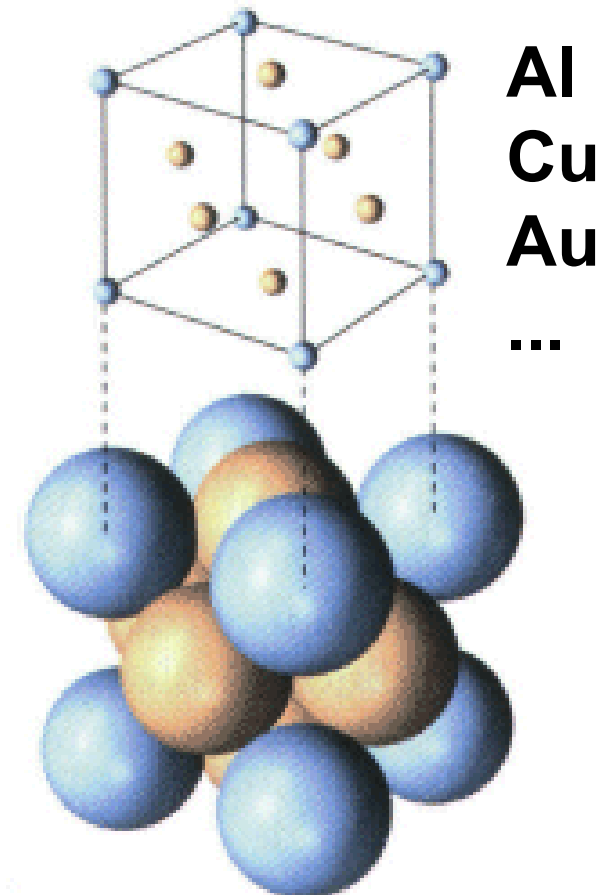
体心立方 BCC

Body-centered cubic



面心立方 FCC

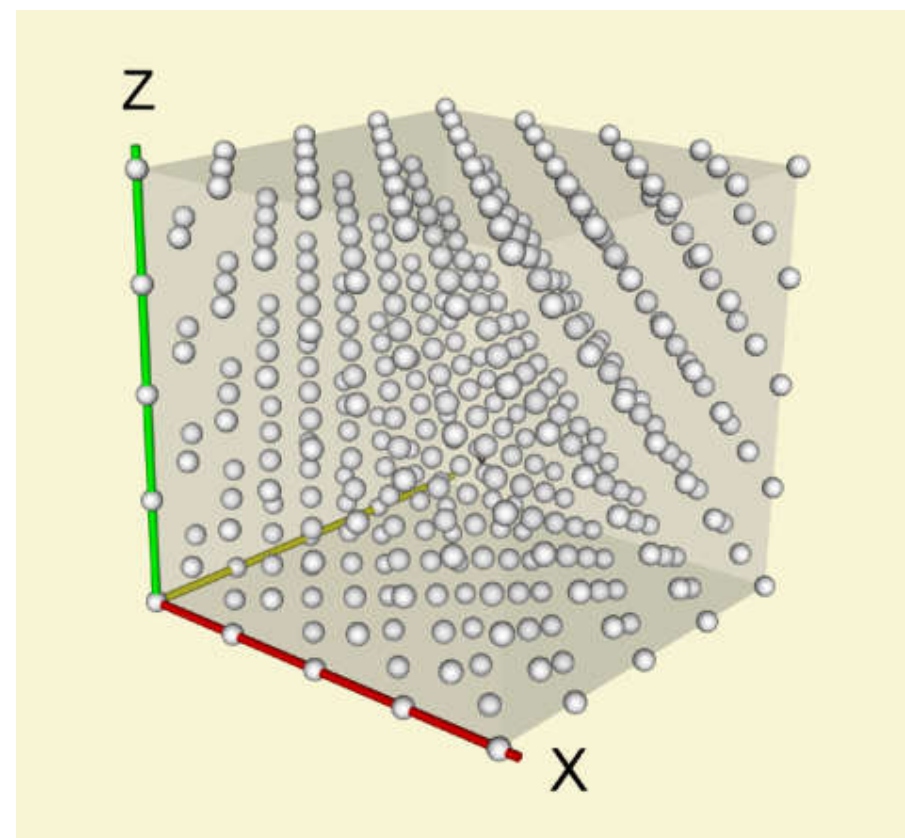
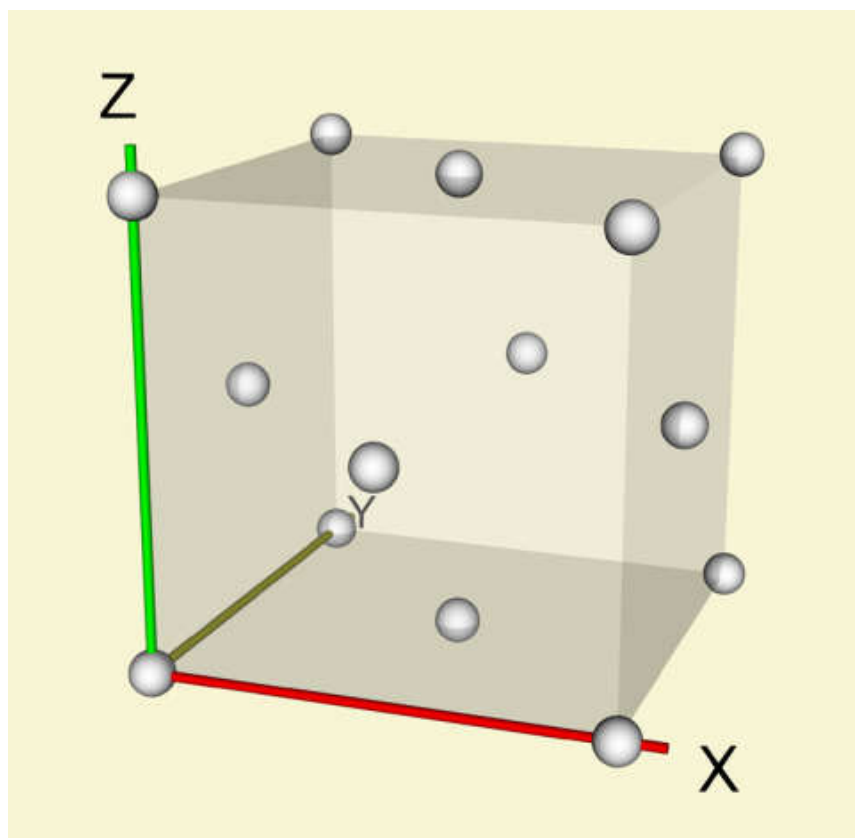
Face-centered cubic



Q: Crystals with SC are not common, why?

FCC is a Bravais Lattice

Each point is *exactly* the same



2D Packing

- **Atomic Packing Factor (APF) 填充因子**
- **APF = (area of circles) / (area of unit cell)**



$$APF = \frac{\pi(0.5a)^2}{a^2} = \frac{\pi}{4}$$

$$= 0.785$$

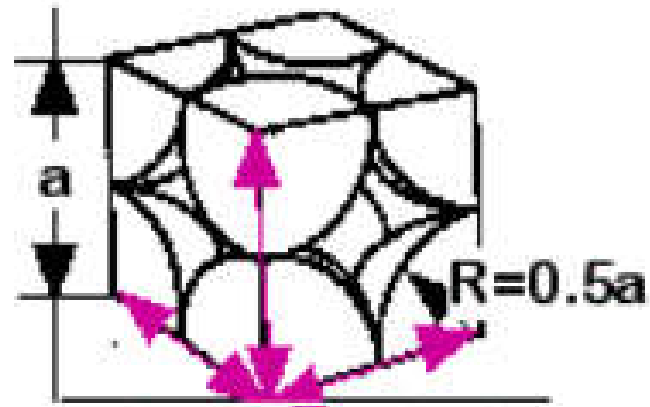
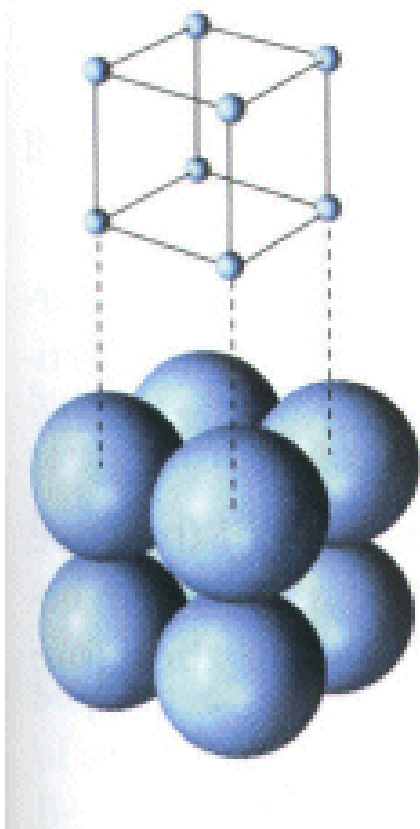
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$$APF = \frac{\pi(0.5a)^2}{a * a * \sqrt{3} / 2} = \frac{\pi}{2\sqrt{3}}$$

$$= 0.906$$

简单立方 SC

- $APF = (\text{volume of spheres}) / (\text{volume of unit cell})$



close-packed
directions

contains $8 \times 1/8 =$
1 atom/unit cell

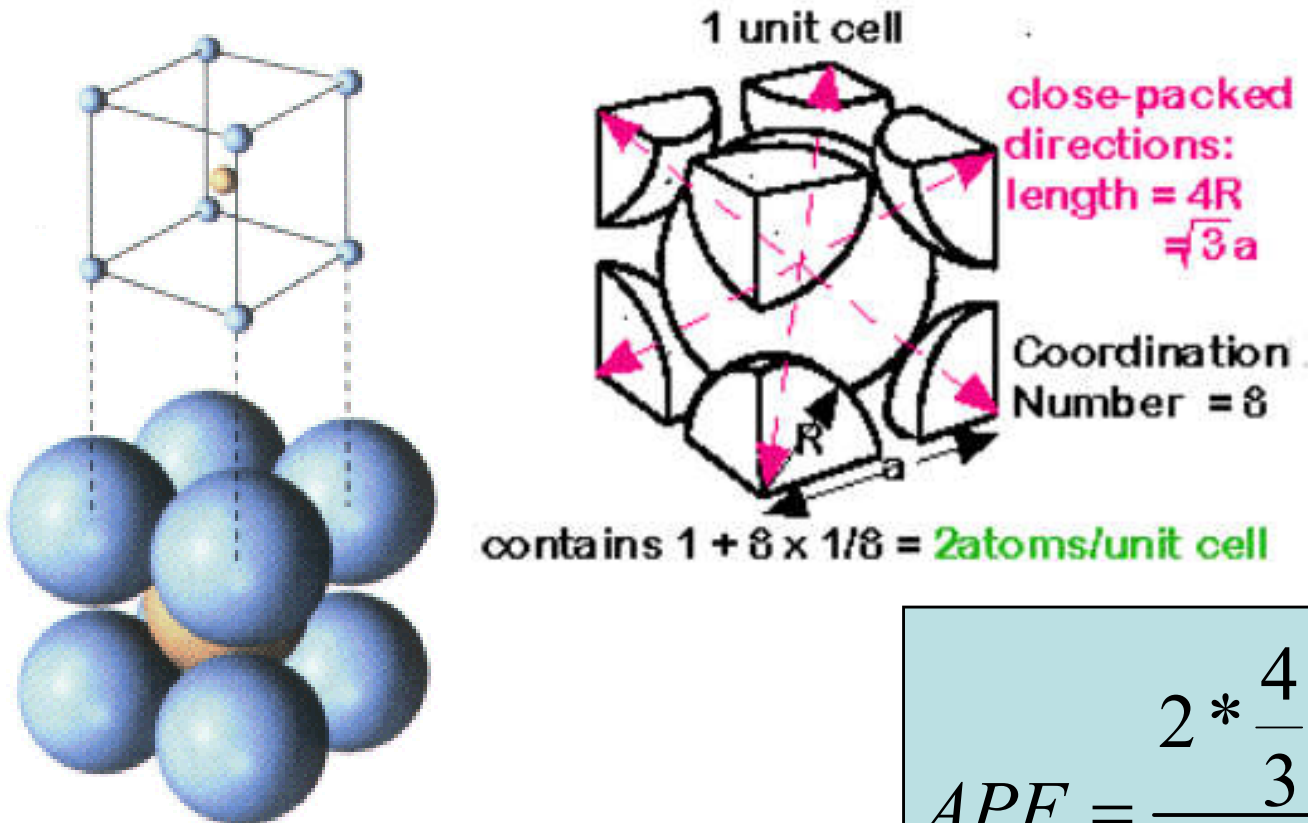
$$APF = \frac{1 * \frac{4}{3} \pi (0.5a)^3}{a^3} = \frac{\pi}{6}$$

$$= 0.523$$

简单立方 SC

体心立方 BCC

- APF = (volume of atoms) / (volume of unit cell)



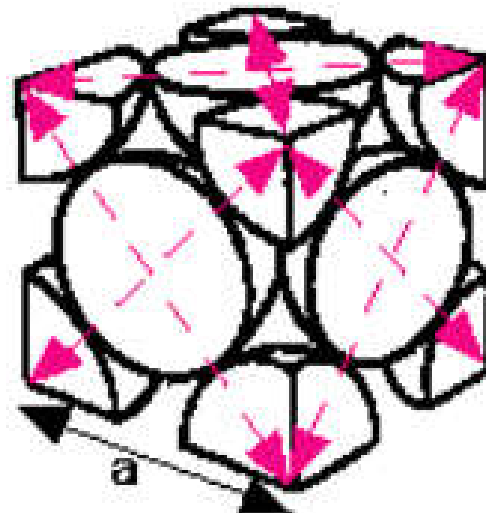
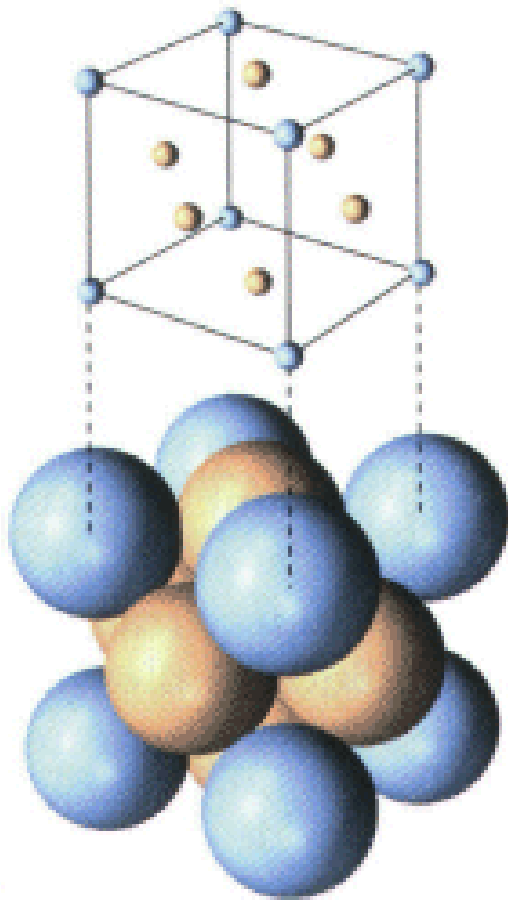
体心立方 BCC

$$APF = \frac{2 * \frac{4}{3} \pi \left(\frac{\sqrt{3}}{4} a\right)^3}{a^3} = \frac{\sqrt{3}}{8} \pi$$

$$= 0.681$$

面心立方 FCC

- APF = (volume of atoms) / (volume of unit cell)



close-packed
directions:
length = $4R$
 $\sqrt{2}a$

Coordination
Number = 12

contains $6 \times 1/2 + 8 \times 1/8 = 4$ atoms/unit cell

$$APF = \frac{4 * \frac{4}{3} \pi \left(\frac{\sqrt{2}}{4} a\right)^3}{a^3} = \frac{\sqrt{2}}{6} \pi$$

$$= 0.740$$

面心立方 FCC

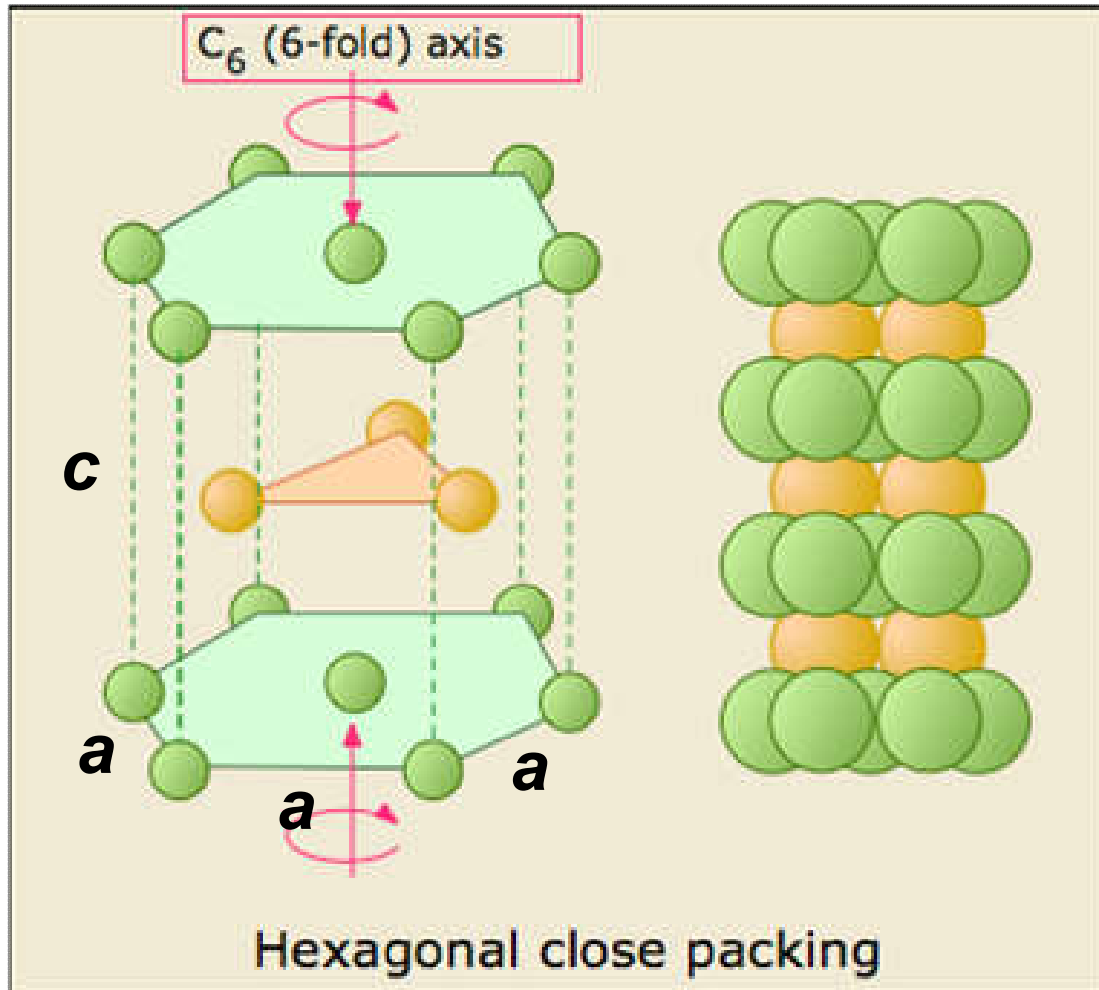
Cubic Lattices

SC BCC FCC

Volume, conventional cell	a^3	a^3	a^3
Lattice points per cell	1	2	4
Volume, primitive cell	a^3	$\frac{1}{2}a^3$	$\frac{1}{4}a^3$
Lattice points per unit volume	$1/a^3$	$2/a^3$	$4/a^3$
Number of nearest neighbors	6	8	12
Nearest-neighbor distance	a	$3^{1/2} a/2 = 0.866a$	$a/2^{1/2} = 0.707a$
Number of second neighbors	12	6	6
Second neighbor distance	$2^{1/2}a$	a	a
Packing fraction ^a	$\frac{1}{6}\pi$ =0.524	$\frac{1}{8}\pi\sqrt{3}$ =0.680	$\frac{1}{6}\pi\sqrt{2}$ =0.740

a: lattice constant / lattice parameter (晶格常数 / 晶格参数)

Hexagonal Close Packing (HCP) 六角密排



Mg, Zn, Ti,...

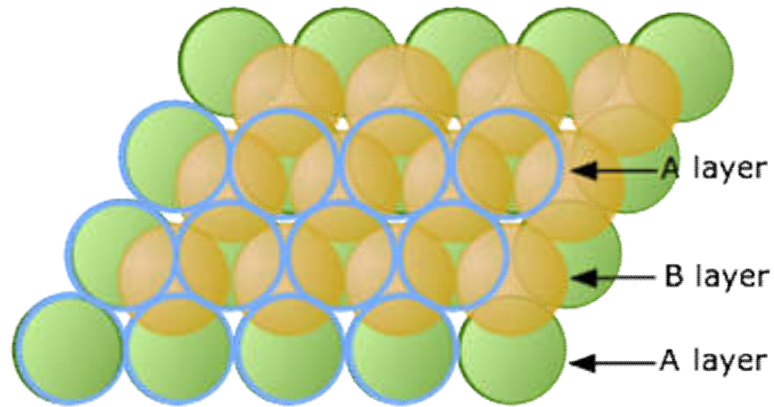
APF for HCP

$$APF = 0.740$$

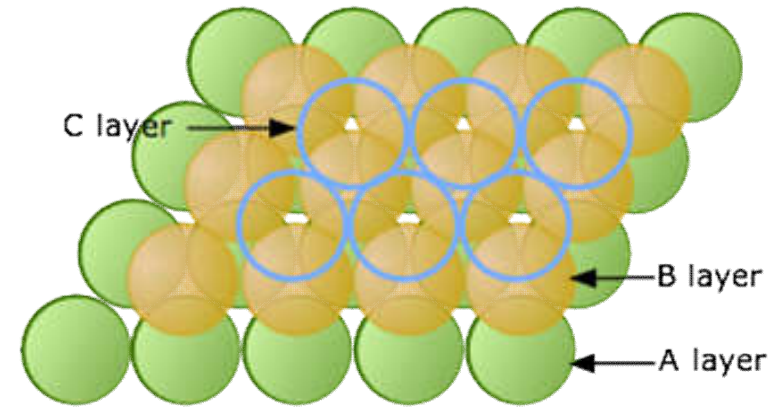
same as FCC

Q: Is HCP a Bravais lattice?
 $c/a = ???$

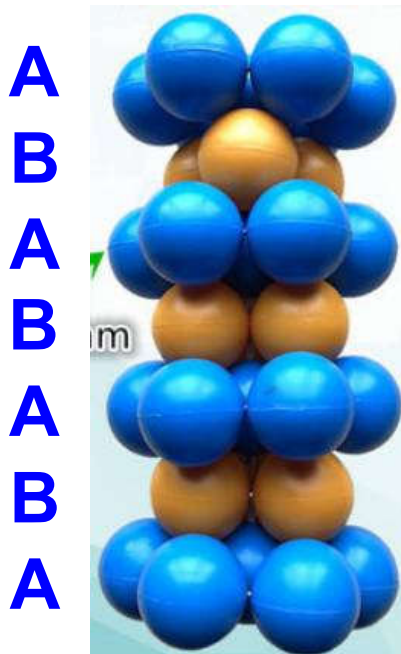
HCP and FCC



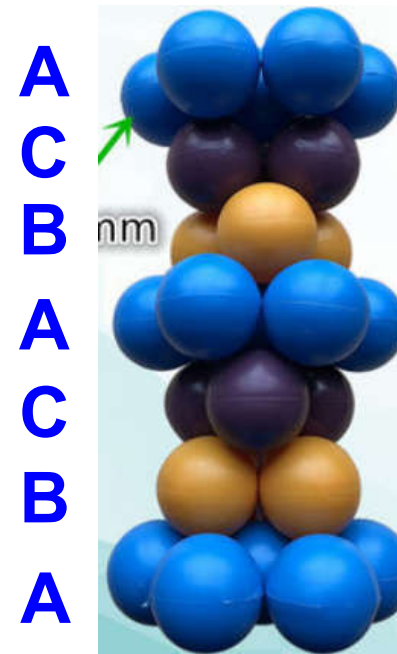
ABA hexagonal close packed



ABC face-centered cubic



HCP



FCC

HCP and FCC - A Little Story



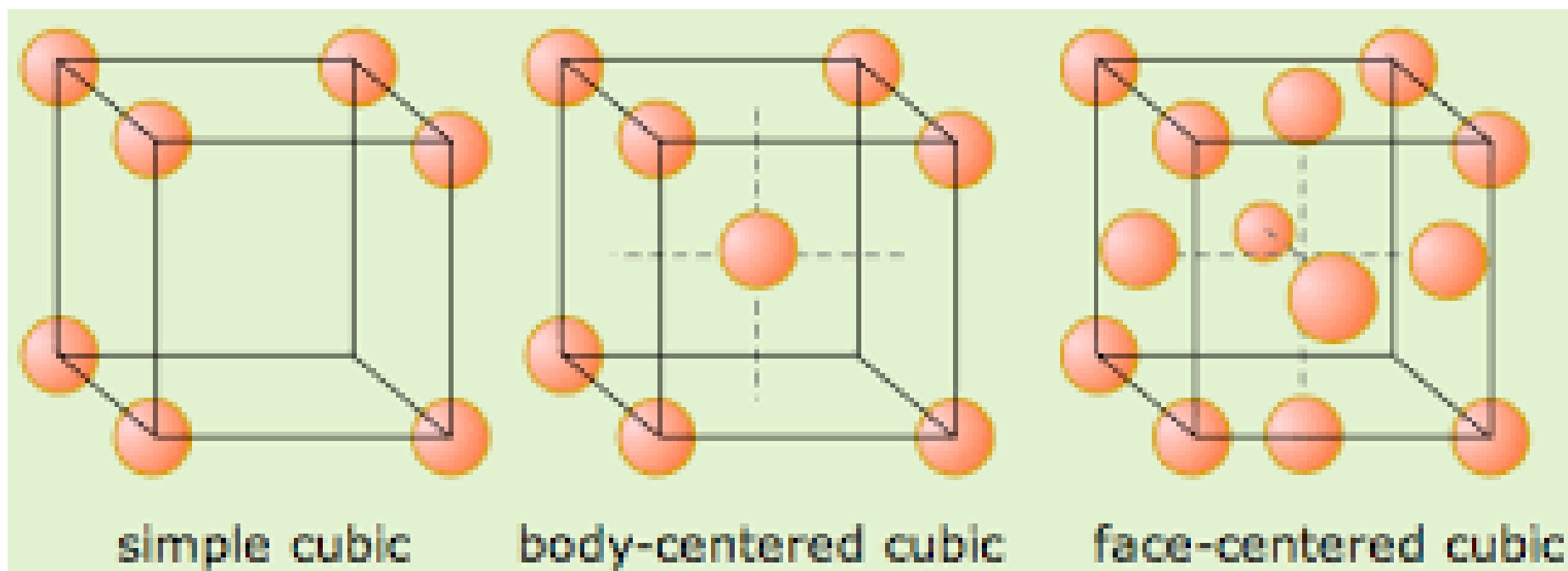
cannonball
pack

Q: FCC, or HCP?

- HCP and FCC are the densest way to pack equal-sized spheres in 3D space
 - J. Kepler's conjecture (开普勒猜想) in 1611
 - Everyone believes that it should be true, but no proof until ...
- T. Hales in Univ. Pittsburgh proved it in 1998

Cubic Lattices

- These are *conventional cells* 惯用晶胞
- They are *not primitive cells* for BCC and FCC. *Why?*



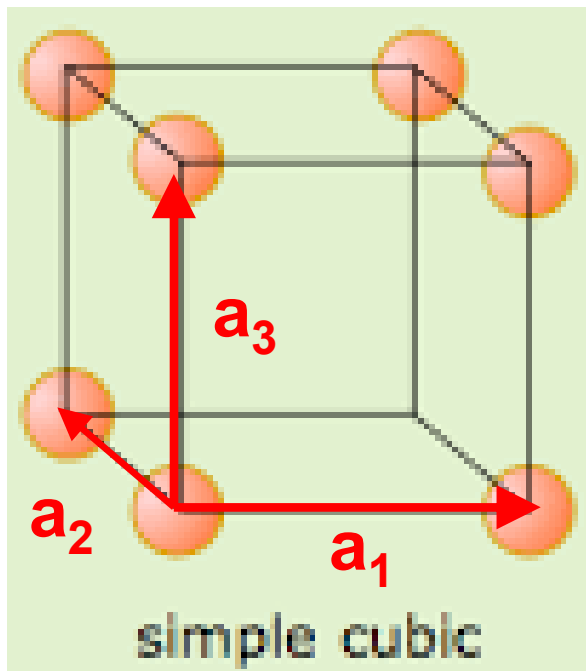
简单立方 SC

体心立方 BCC

面心立方 FCC

Q: How to draw primitive cells for BCC and FCC?

A Primitive Cell for SC

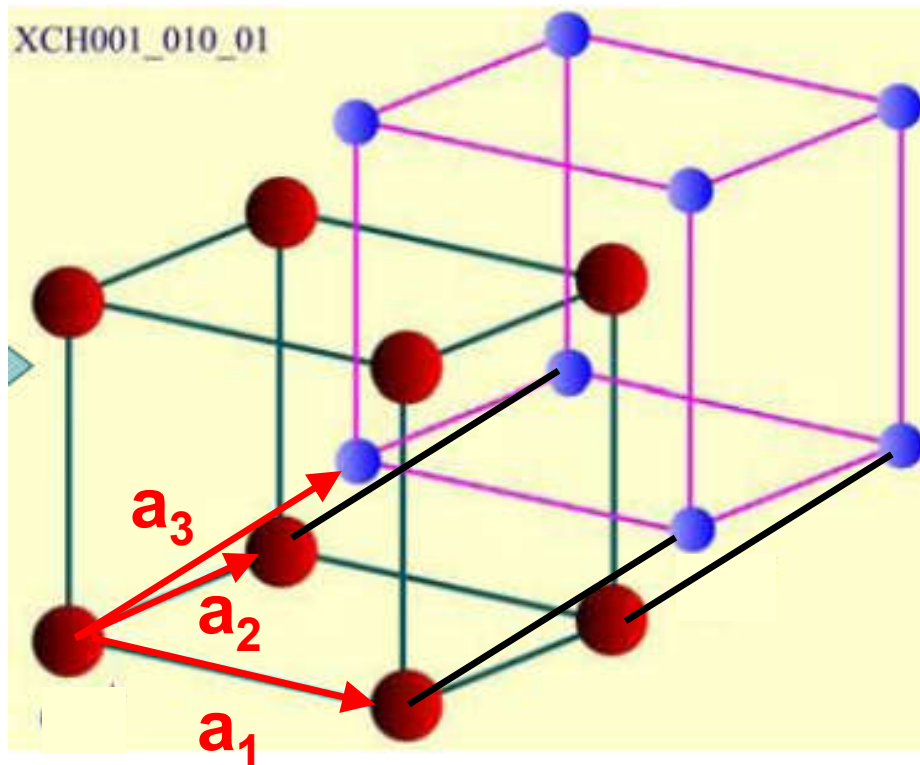


$$\mathbf{a}_1 = a\hat{\mathbf{x}}$$
$$\mathbf{a}_2 = a\hat{\mathbf{y}}$$
$$\mathbf{a}_3 = a\hat{\mathbf{z}}$$

$(\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}})$ are Cartesian coordinates

The volume of this primitive cell = a^3

A Primitive Cell for BCC



$$\mathbf{a}_1 = a\hat{\mathbf{x}}$$

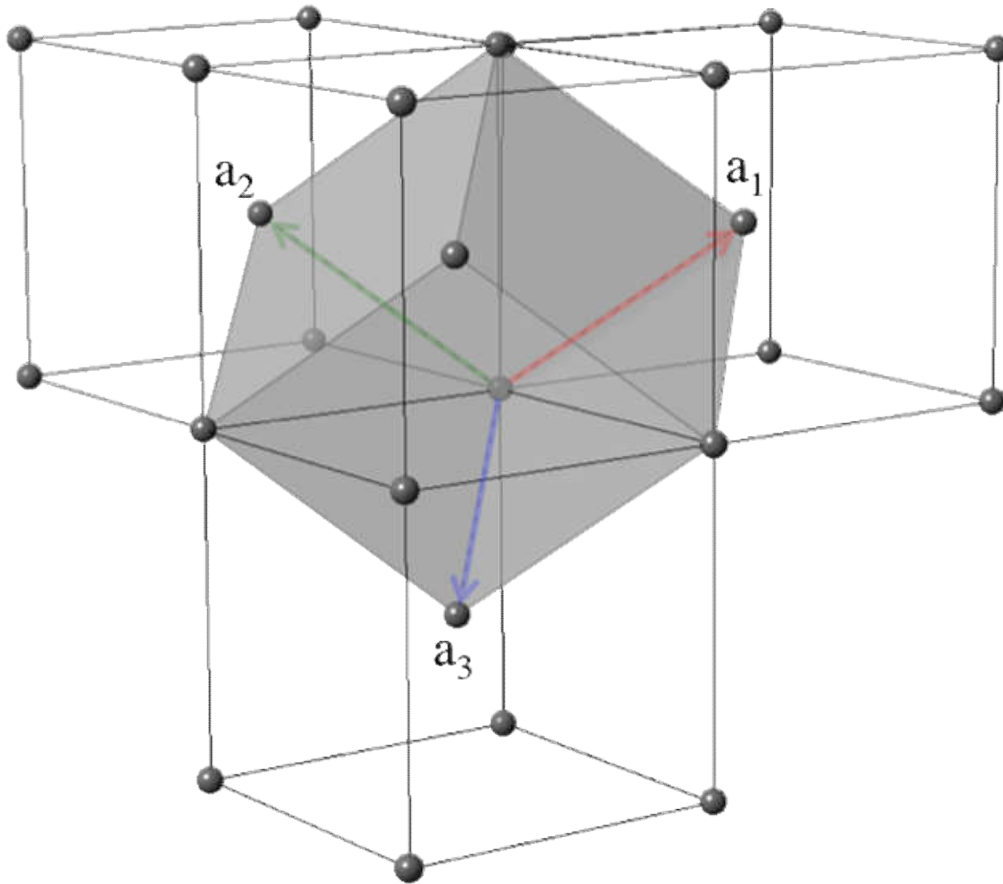
$$\mathbf{a}_2 = a\hat{\mathbf{y}}$$

$$\mathbf{a}_3 = \frac{a}{2}(\hat{\mathbf{x}} + \hat{\mathbf{y}} + \hat{\mathbf{z}})$$

Q: What is the volume of the primitive cell?

Another Primitive Cell for BCC

a more symmetric one



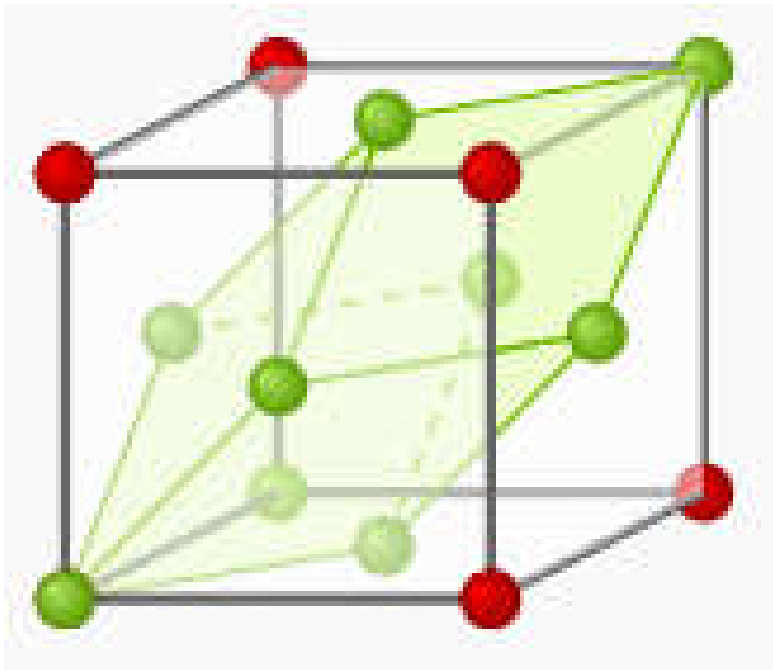
$$\mathbf{a}_1 = \frac{a}{2} (-\hat{\mathbf{x}} + \hat{\mathbf{y}} + \hat{\mathbf{z}})$$

$$\mathbf{a}_2 = \frac{a}{2} (\hat{\mathbf{x}} - \hat{\mathbf{y}} + \hat{\mathbf{z}})$$

$$\mathbf{a}_3 = \frac{a}{2} (\hat{\mathbf{x}} + \hat{\mathbf{y}} - \hat{\mathbf{z}})$$

Q: What is the volume of the primitive cell?

A Primitive Cell for FCC



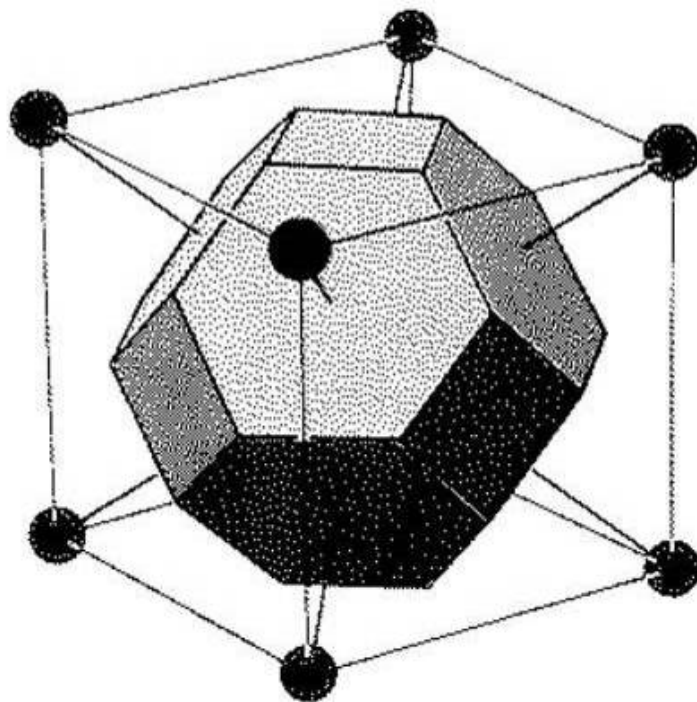
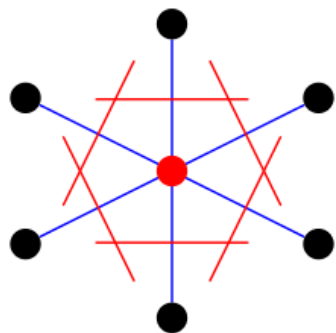
$$\mathbf{a}_1 = \frac{a}{2} (\hat{\mathbf{y}} + \hat{\mathbf{z}})$$

$$\mathbf{a}_2 = \frac{a}{2} (\hat{\mathbf{z}} + \hat{\mathbf{x}})$$

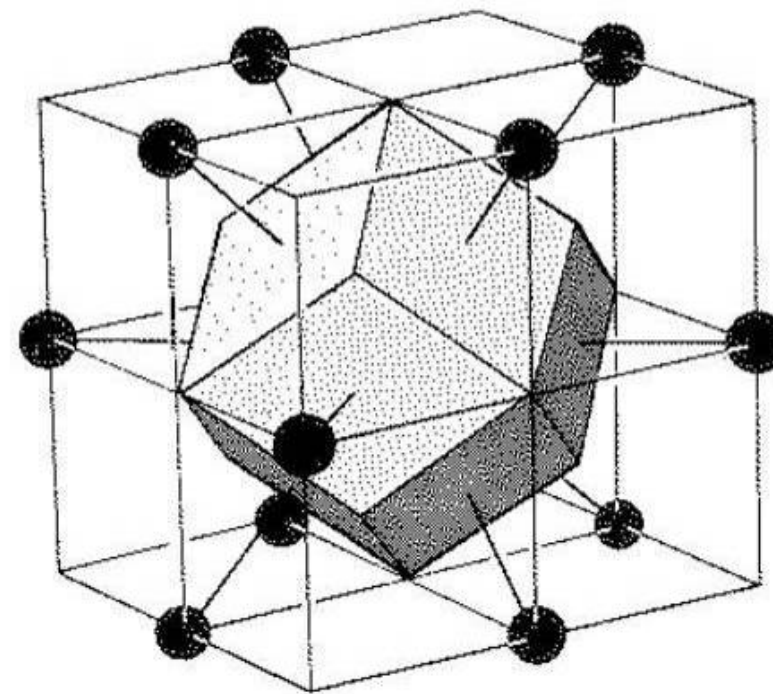
$$\mathbf{a}_3 = \frac{a}{2} (\hat{\mathbf{x}} + \hat{\mathbf{y}})$$

Q: What is the volume of the primitive cell?

Wigner-Seitz cells for BCC and FCC



体心立方 BCC

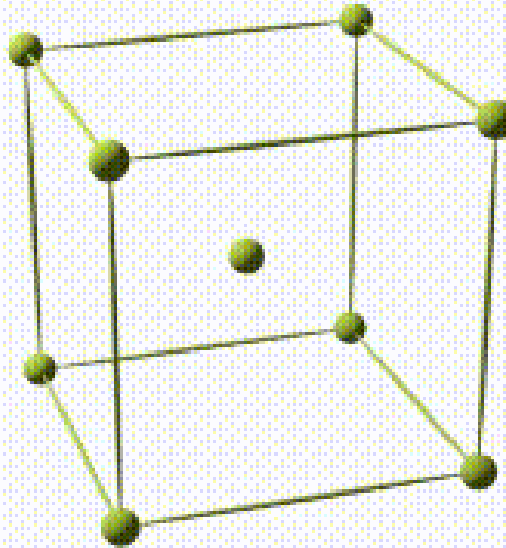


面心立方 FCC

Q: What is the volume of the Wigner-Seitz cell?

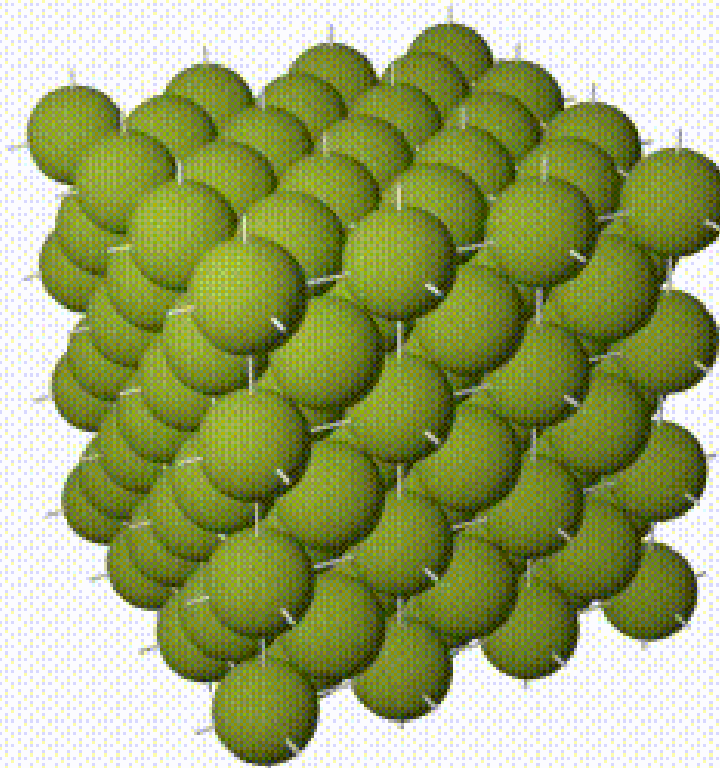
Review: BCC and FCC

The bcc structure



Review: BCC and FCC

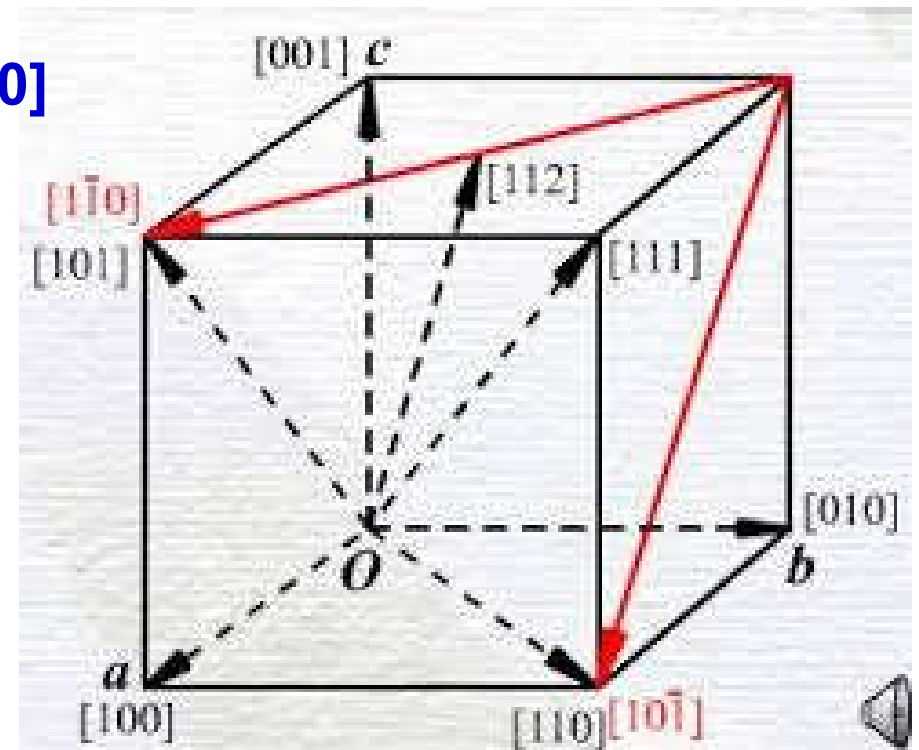
The fcc structure



Miller Indices - Direction 晶向

- crystal direction $[hkl]$
 - in cubic lattices, $[hkl]$ direction \perp (hkl) plane
- $\langle hkl \rangle$
 - a group of similar directions
 - $\langle 001 \rangle$ includes $[001]$, $[010]$, $[100]$

h, k, l are integers with
no common factors

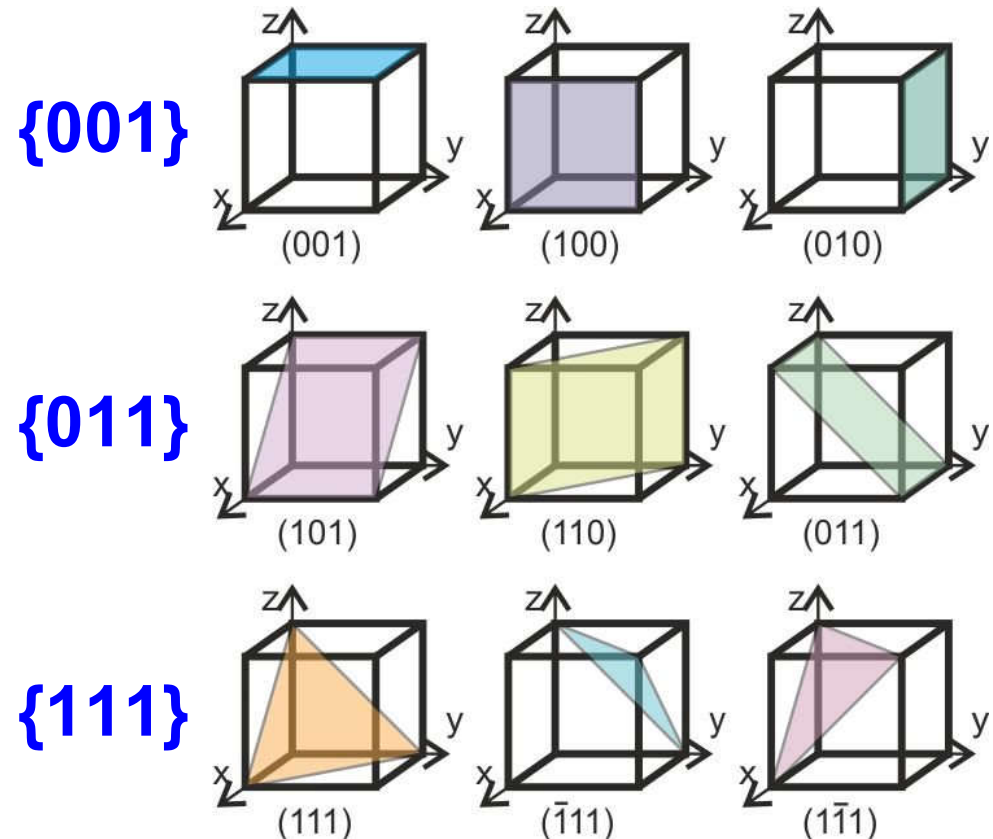


Miller Indices - Plane 晶面

- crystal plane (hkl)
 - intercepts at $(a_1/h, a_2/k, a_3/l)$
- $\{hkl\}$
 - a group of similar planes

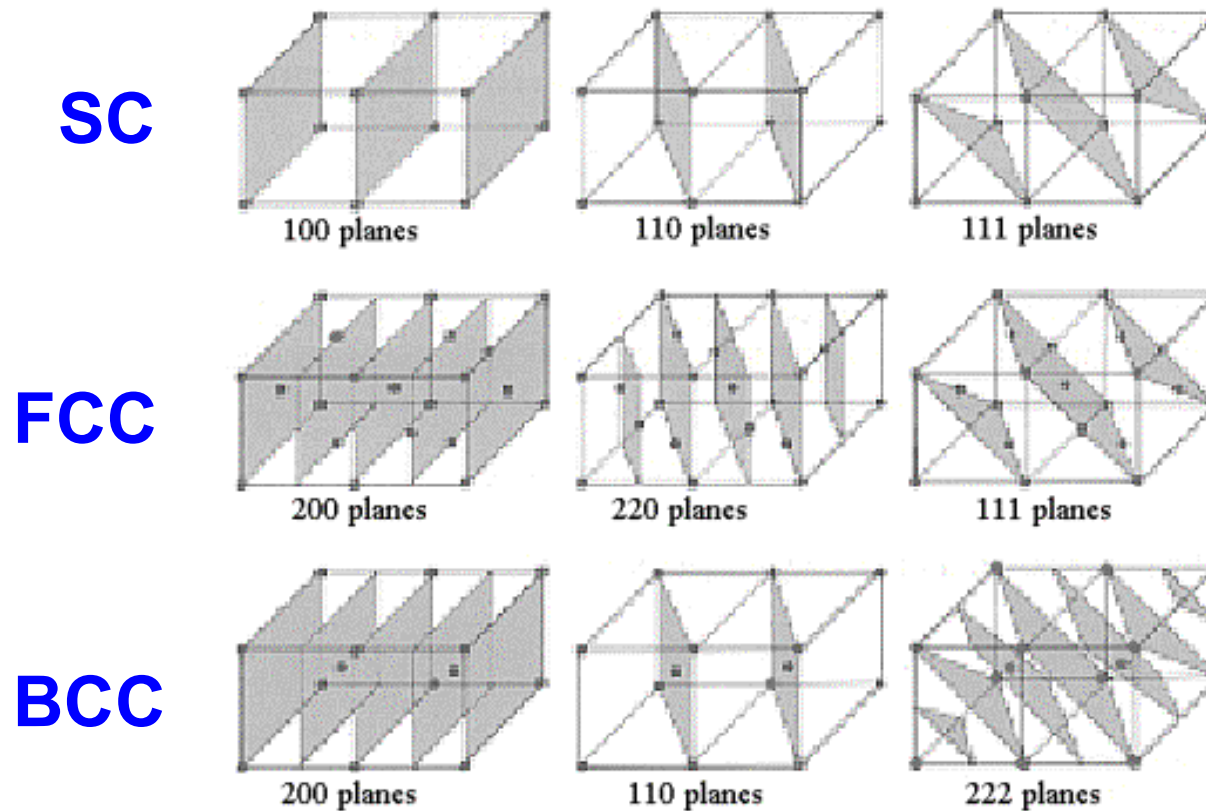
h, k, l are integers with
no common factors

**Q: Draw planes:
(120), (112), (131)**



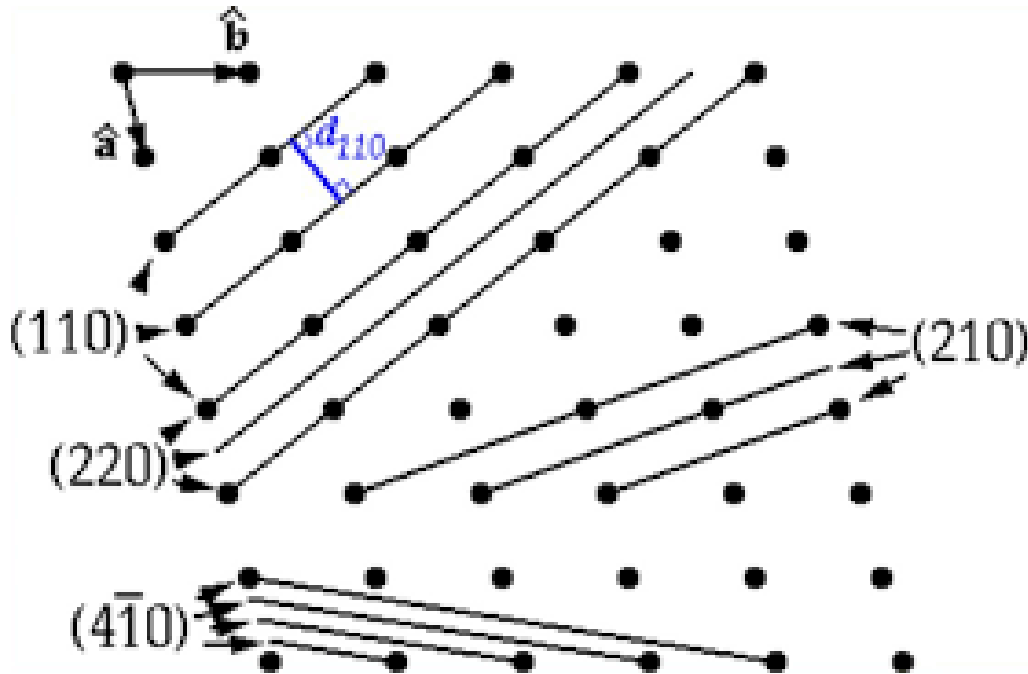
Lattice Plane 晶面

- any plane with at least 3 noncollinear Bravais lattice points
 - not always planes with Miller index, e.g., (200), (220), (222)



Interplanar Spacing 晶面间距

- Distance between adjacent lattice planes



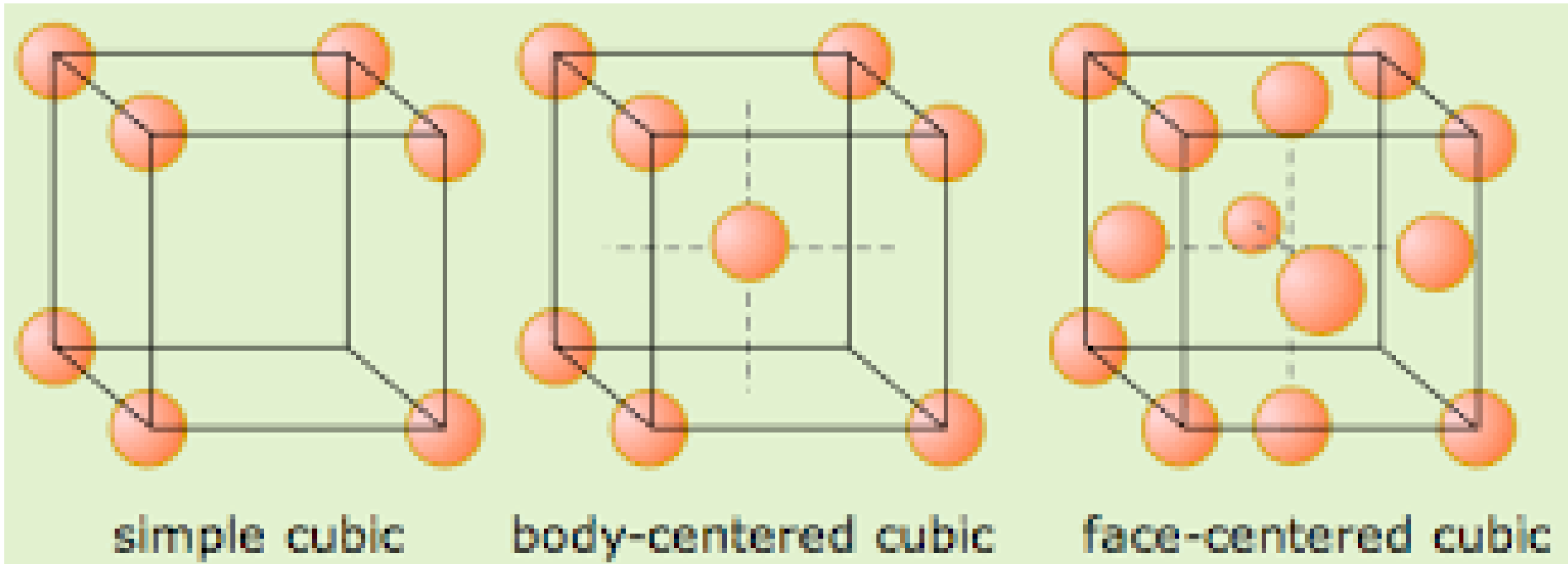
For cubic systems
(SC, BCC, FCC)

$$d_{(hkl)} = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

**Q: Interplanar Spacing for
(120), (112), (131)**

Coordination Number 配位数

- The number of the nearest neighbors
 - 'kissing' number



简单立方 SC

6

体心立方 BCC

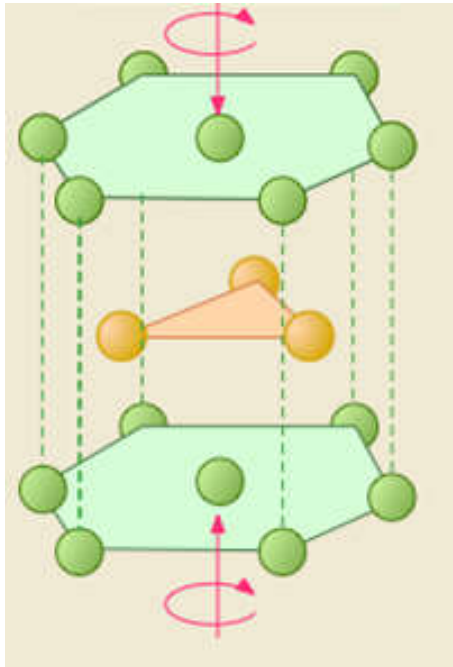
8

面心立方 FCC

12

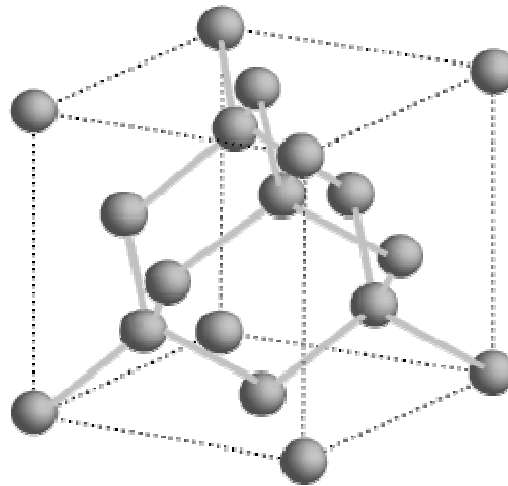
Coordination Number 配位数

- The number of the nearest neighbors
 - 'kissing' number



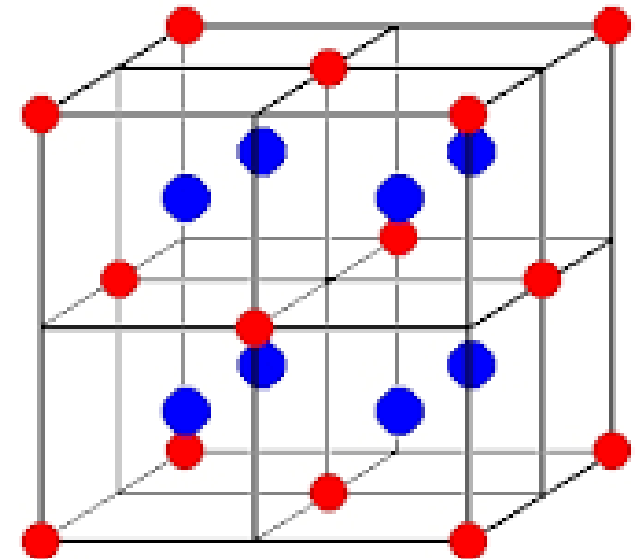
HCP

?



diamond

?



CaF₂

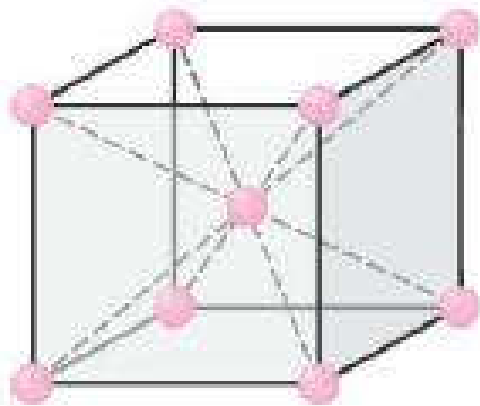
? and ?

Common Crystal Structures

- BCC, FCC, HCP
- Diamond 金刚石
 - C, Si, Ge
- Zinc Blende 闪锌矿
 - GaAs, InP, ZnS
- Halite / Rock Salt 岩盐
 - NaCl, KCl
- CsCl
- Fluorite 萤石
 - CaF₂
- Wurtzite 纤锌矿
 - GaN, ZnO
- Perovskite 钙钛矿
 - CaTiO₃, CsPbBr₃
- 1D, 2D structures
- ...

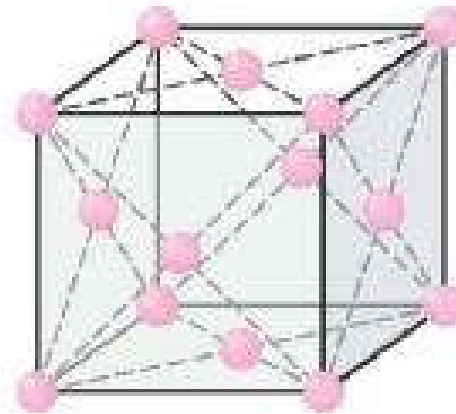
BCC, FCC, HCP

Li, Na, Cr, ...



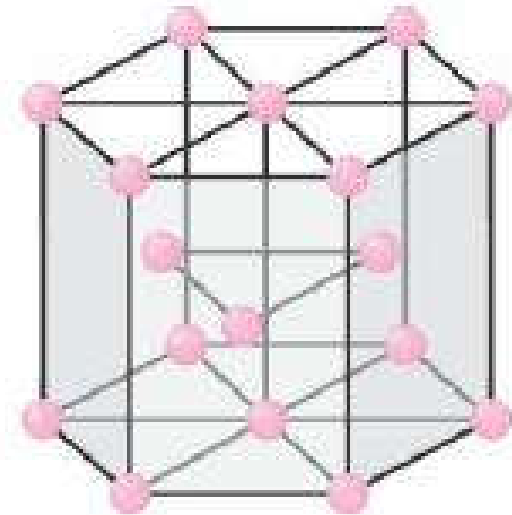
BCC

Al, Cu, Au, ...



FCC

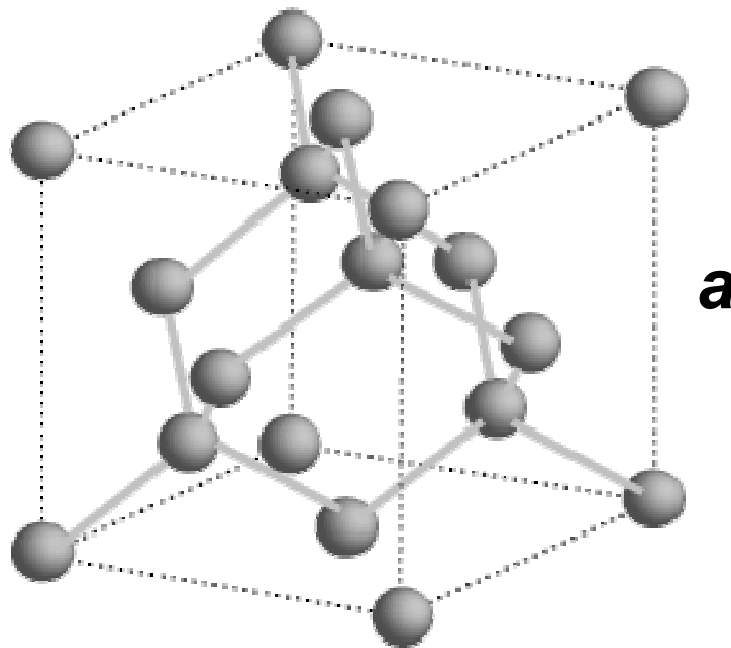
Mg, Zn, Ti, ...



HCP

Diamond Structure

- C, Si, Ge, ...



					2
	5	6	7	8	9
B	C	N	O	F	Ne
13	14	15	16	17	18
Al	Si	P	S	Cl	Ar
31	32	33	34	35	36
Ga	Ge	As	Se	Br	Kr
49	50	51	52	53	54
In	Sn	Sb	Te	I	Xe
81	82	83	84	85	86
Tl	Pb	Bi	Po	At	Rn

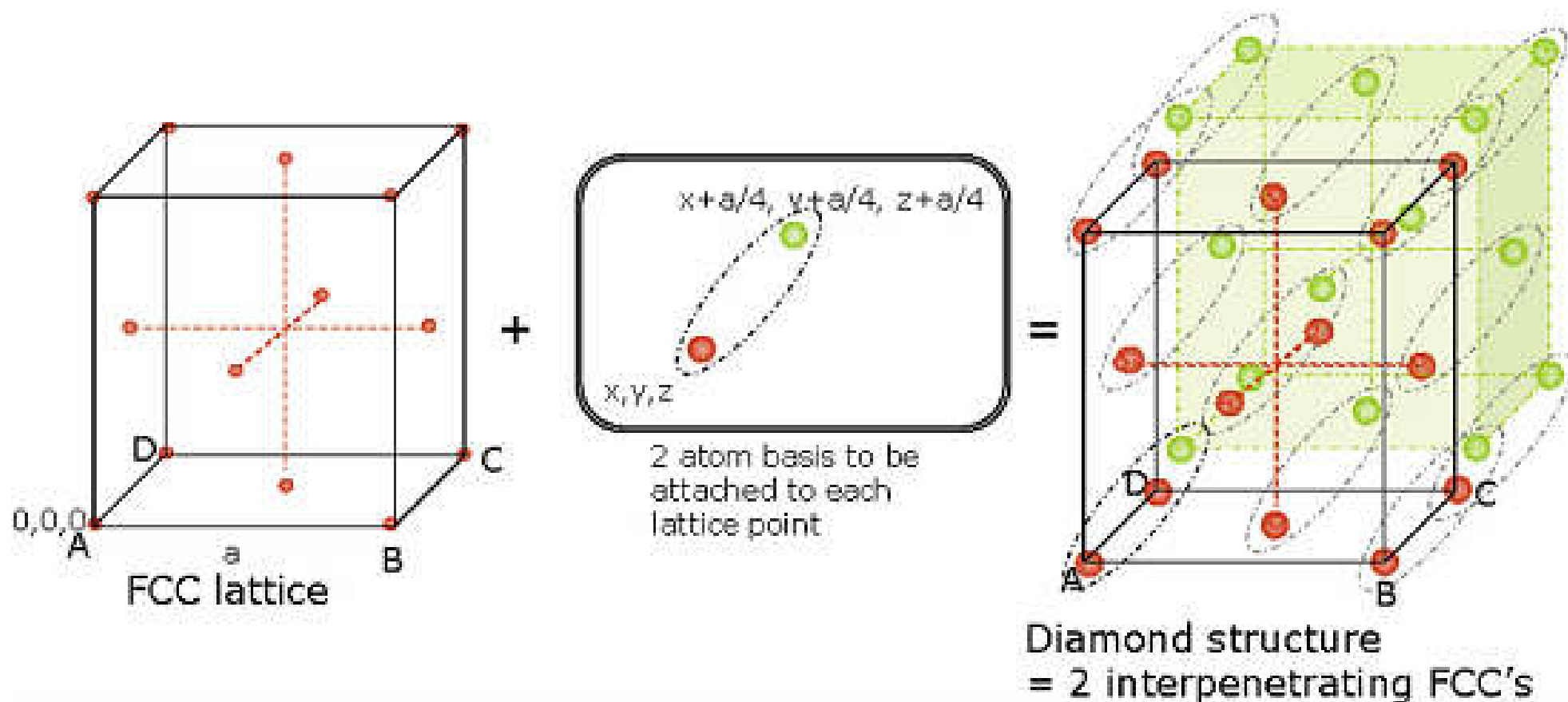
lattice parameters for
diamond structures

	a (Å)
C (diamond)	3.57
Si	5.43
Ge	5.66
α -Sn	6.49

Q:
What is the Bravais lattice?
What is the atomic distance?
How many atoms in the cubic cell?
What is the APF?

Diamond Structure

- C, Si, Ge, ...



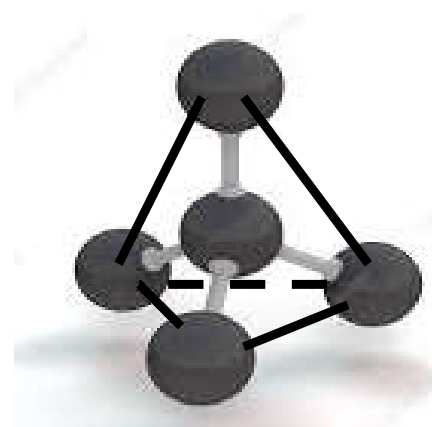
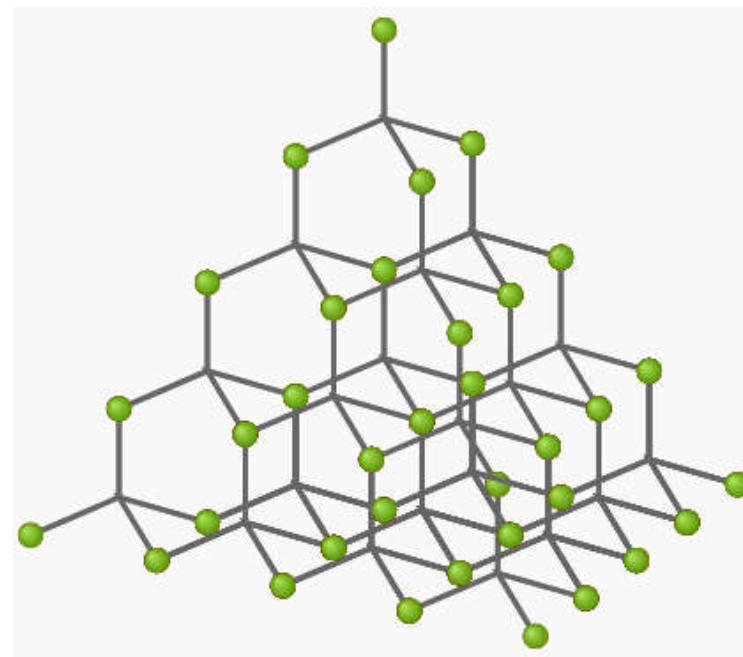
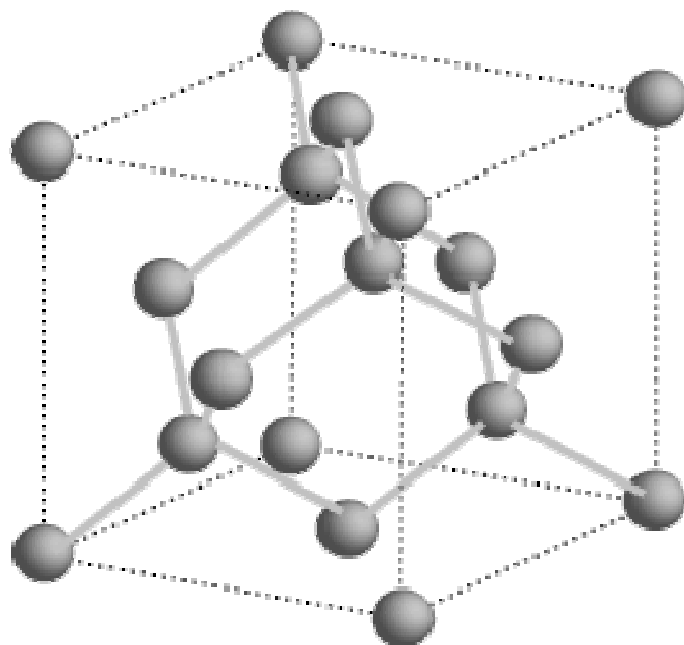
Lattice

Basis

Crystal

Diamond Structure

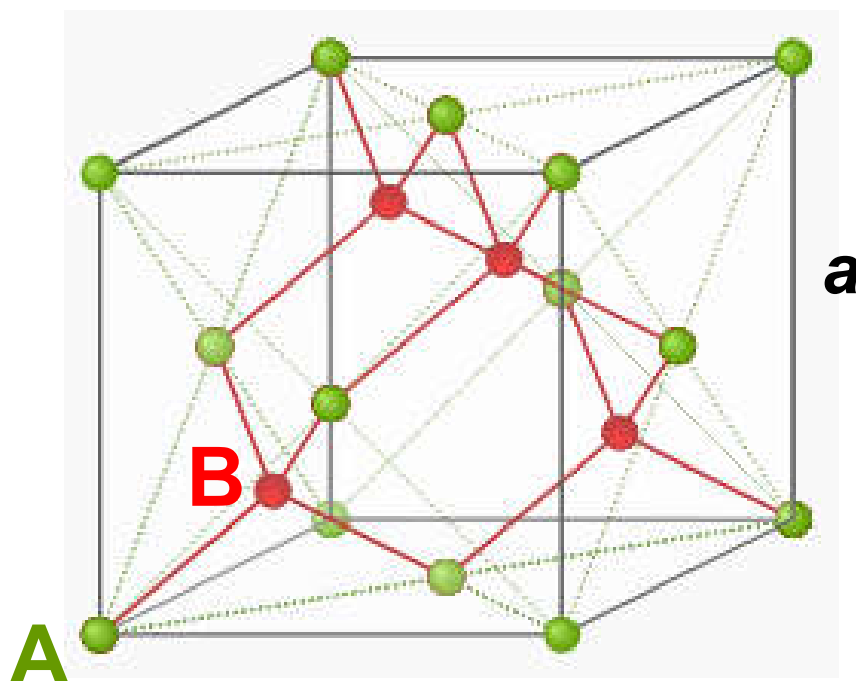
- C, Si, Ge, ...



tetrahedron
(正四面体)

Zinc Blende 闪锌矿

- GaAs, InP, ZnS, ...



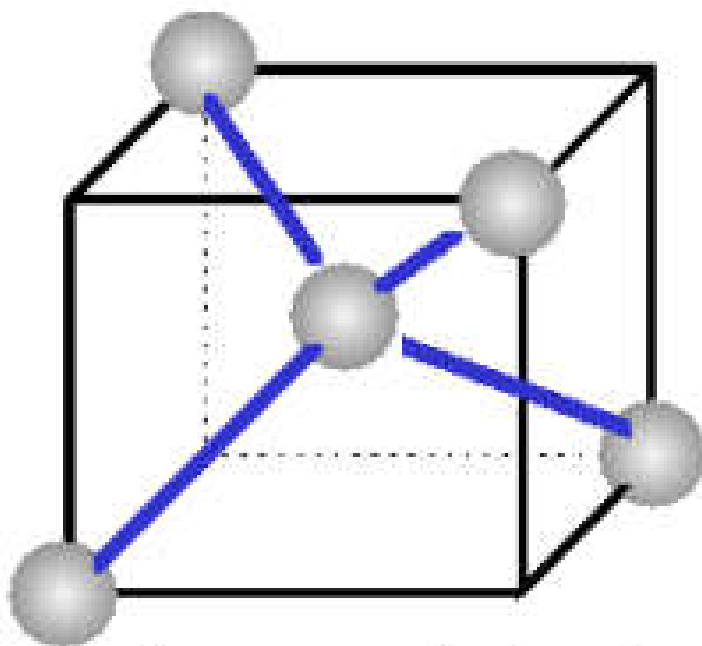
					2
					He
5	6	7	8	9	10
B	C	N	O	F	Ne
13	14	15	16	17	18
Al	Si	P	S	Cl	Ar
31	32	33	34	35	36
Ga	Ge	As	Se	Br	Kr
49	50	51	52	53	54
In	Sn	Sb	Te	I	Xe
81	82	83	84	85	86
Tl	Pb	Bi	Po	At	Rn

Q:

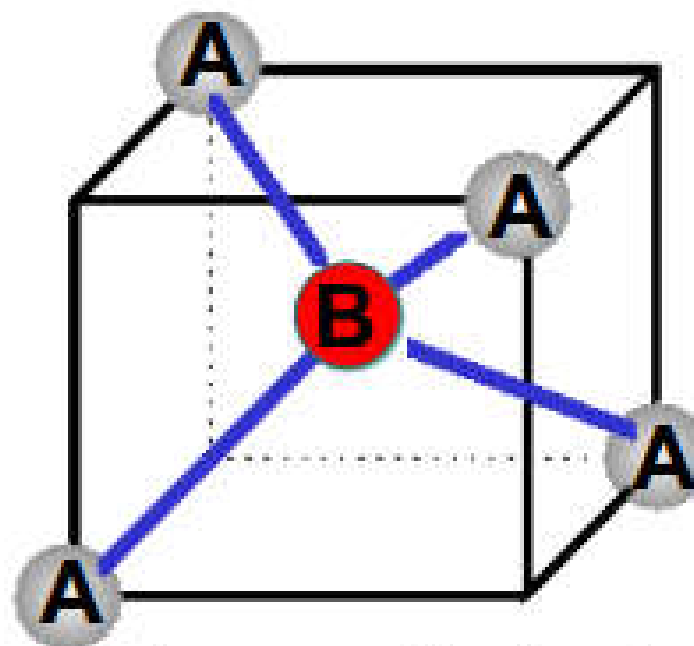
How many A and B atoms in the cubic cell?

Diamond vs. Zinc Blende

- FCC lattice



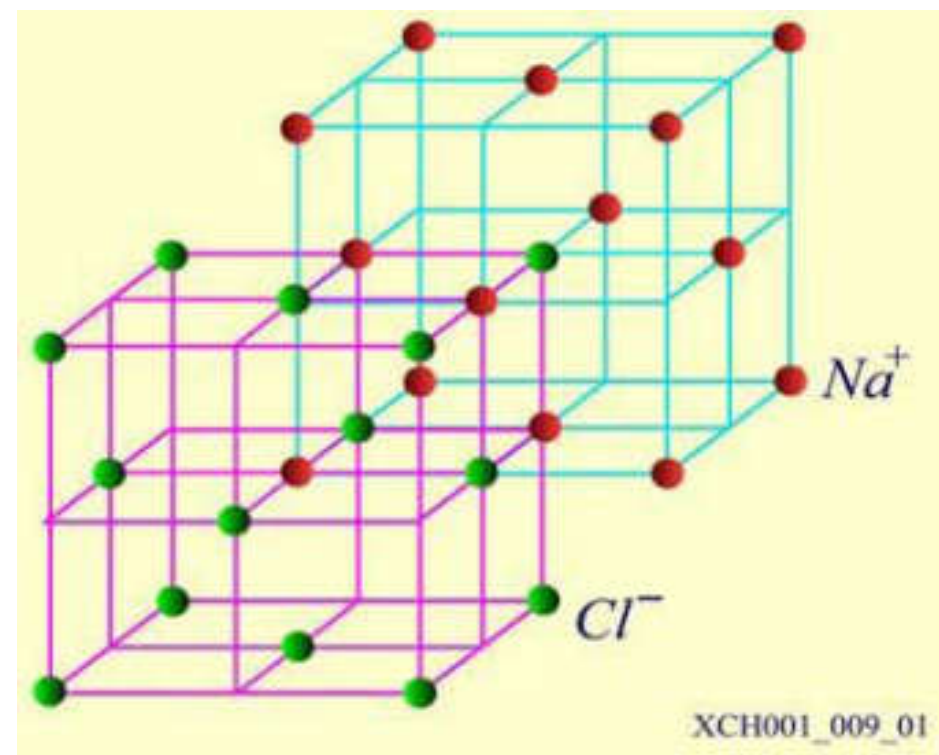
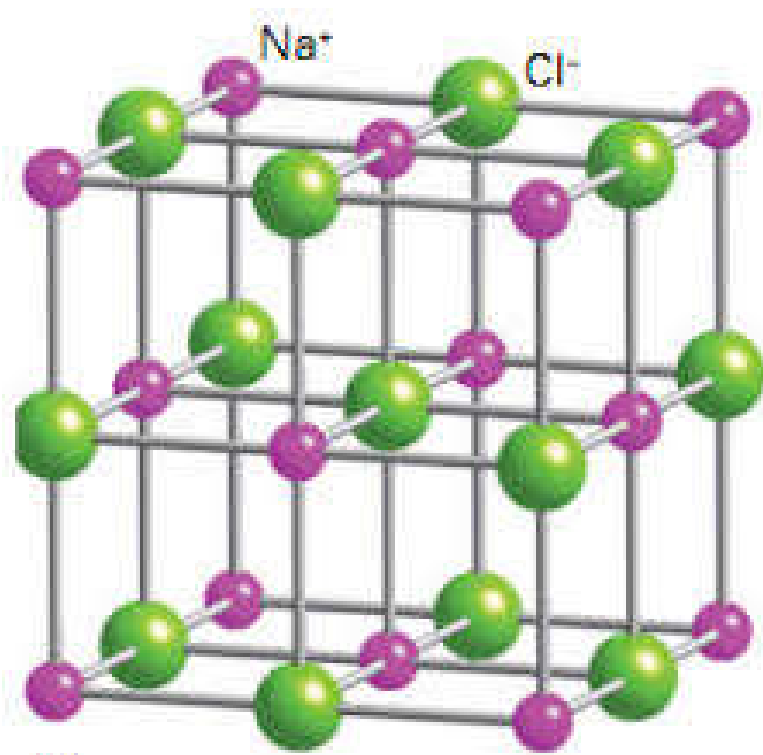
Si



GaAs

Halite / Rock Salt 岩盐

- NaCl, ...



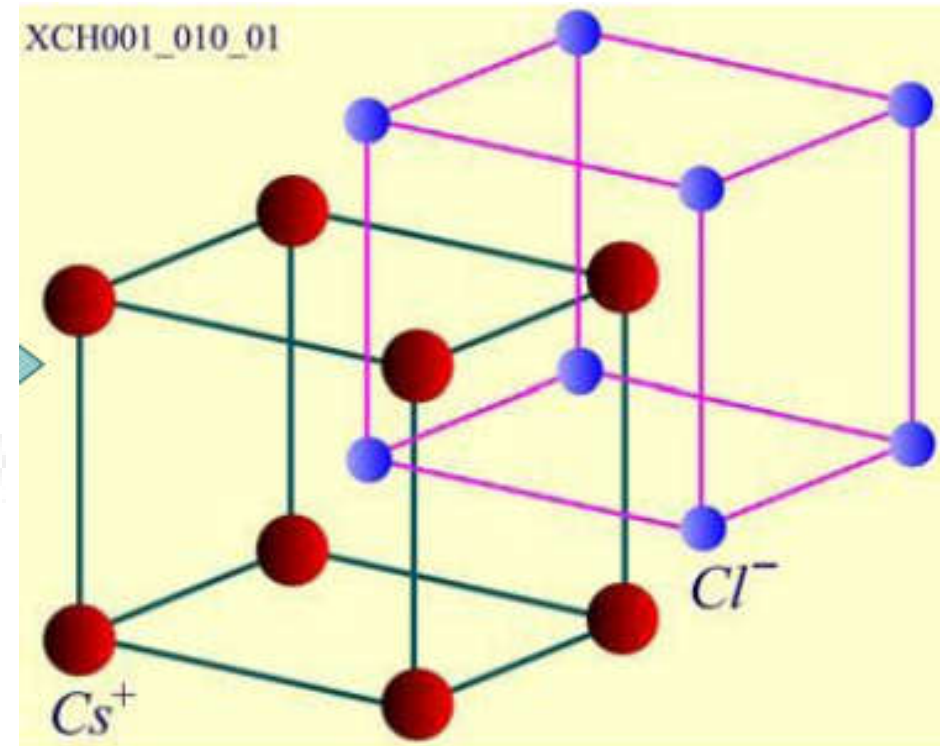
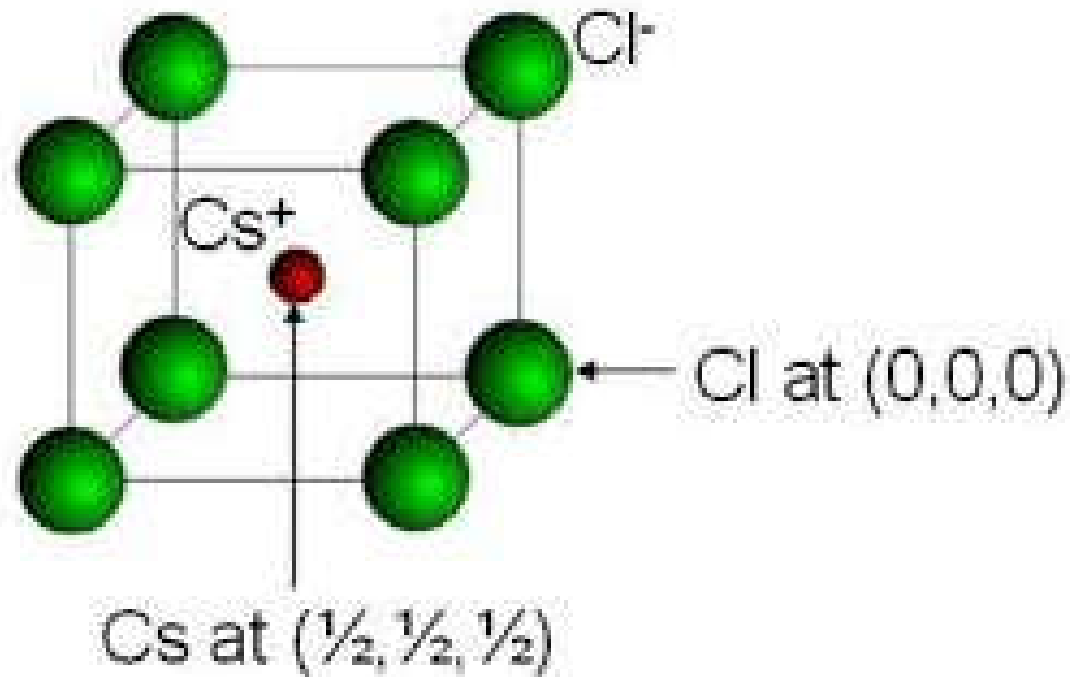
Q:

What is the Bravais lattice?

How many A and B atoms in the cubic cell?

CsCl

- CsCl, ...



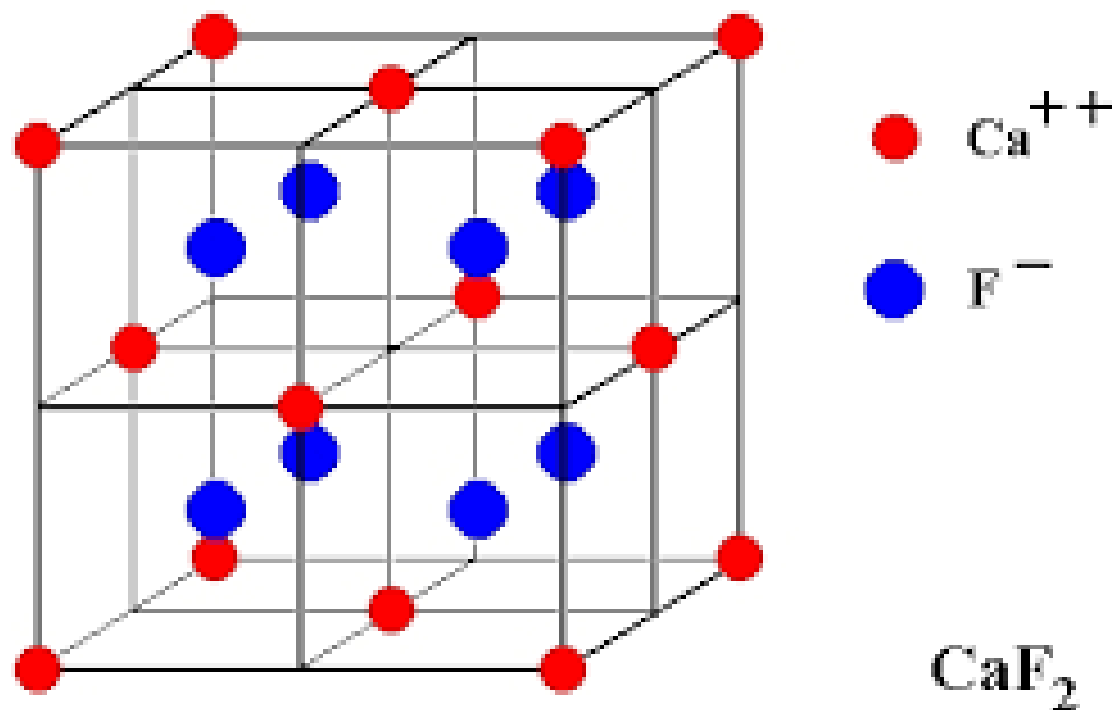
Q:

What is the Bravais lattice?

How many A and B atoms in the cubic cell?

Fluorite 萤石

- CaF_2 , ...



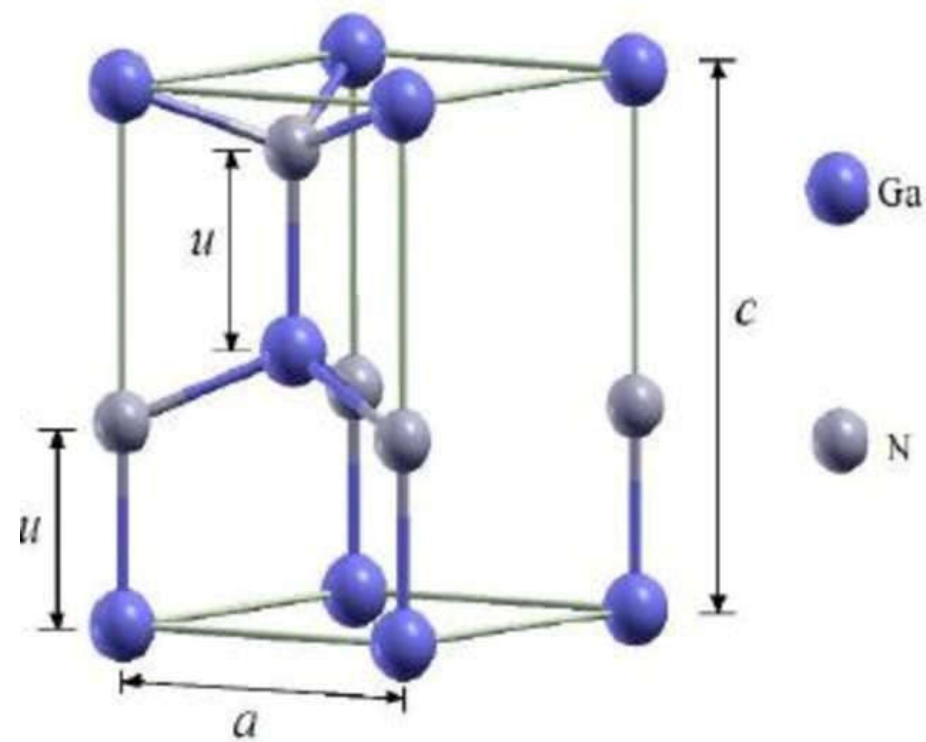
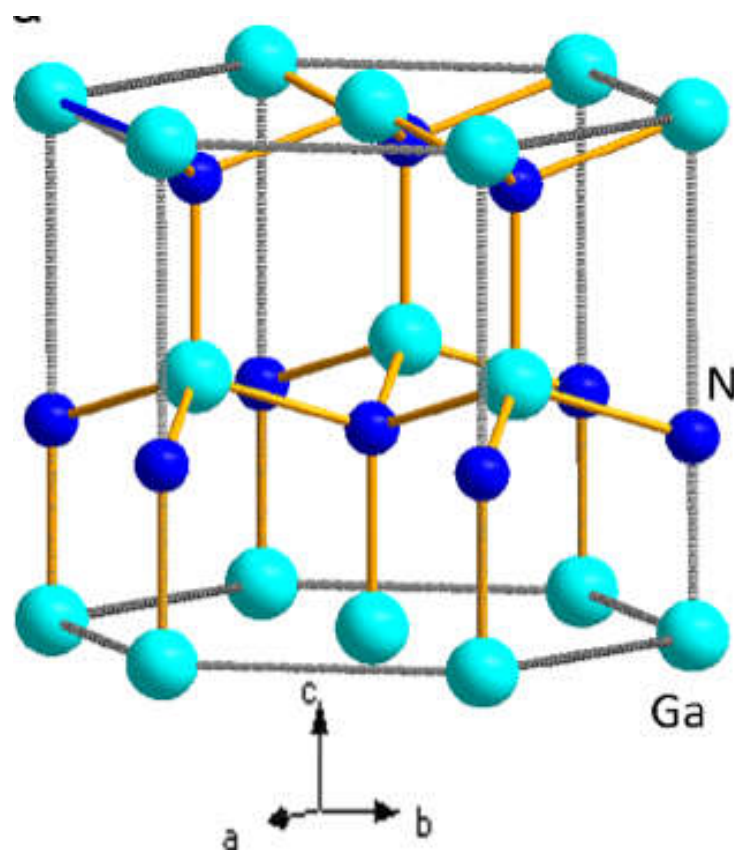
Q:

What is the Bravais lattice?

How many Ca and F atoms in the cubic cell?

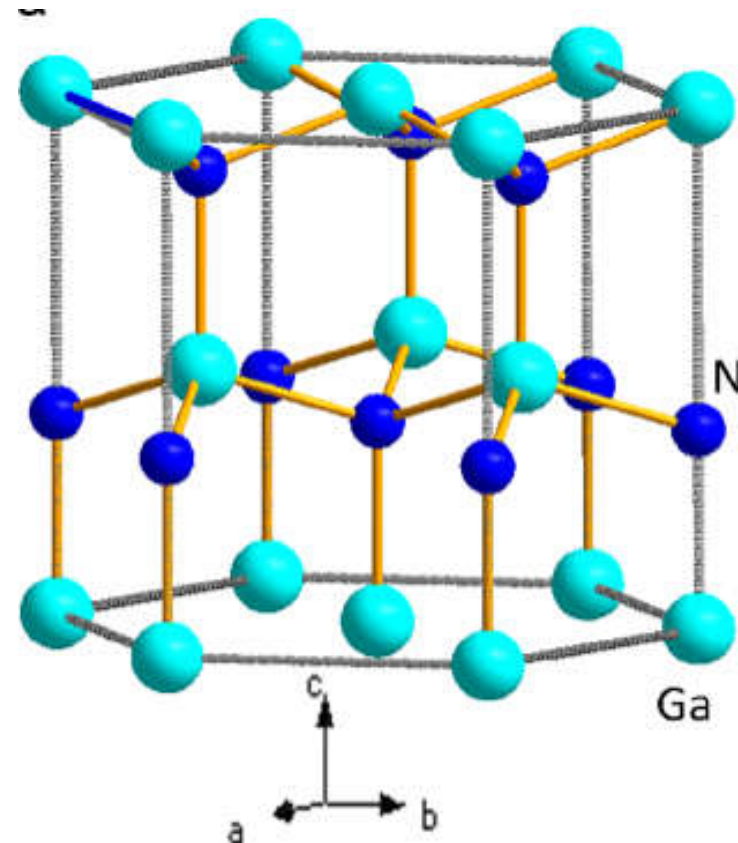
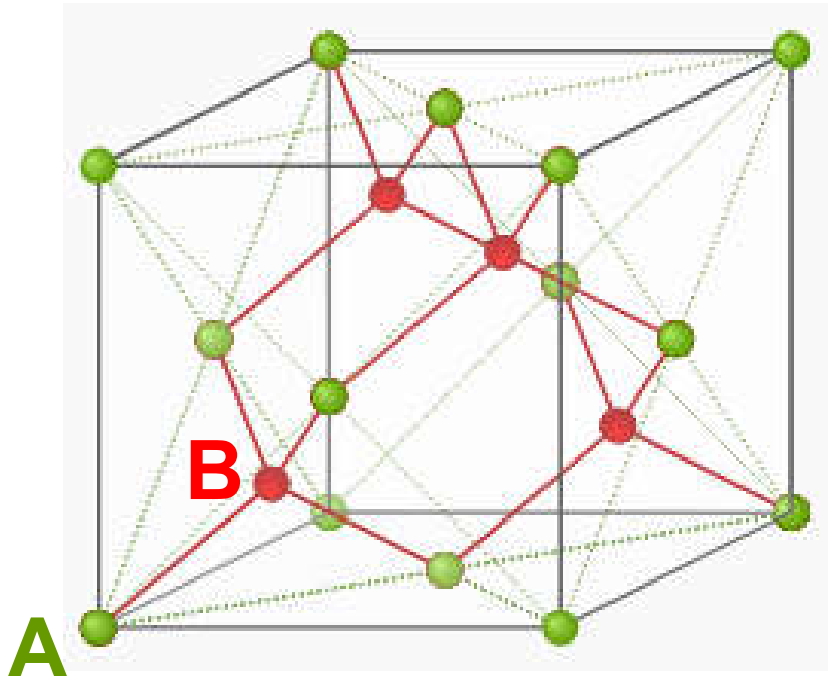
Wurtzite 纤锌矿

- GaN, ZnO, ...



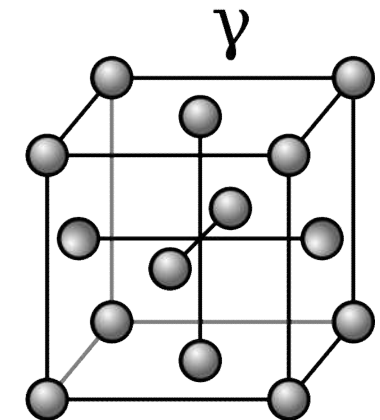
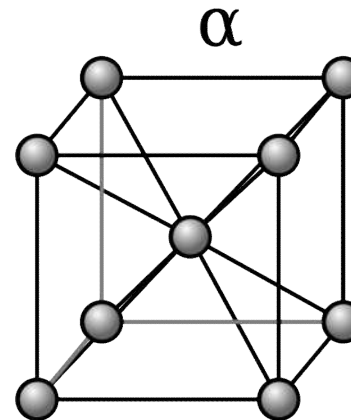
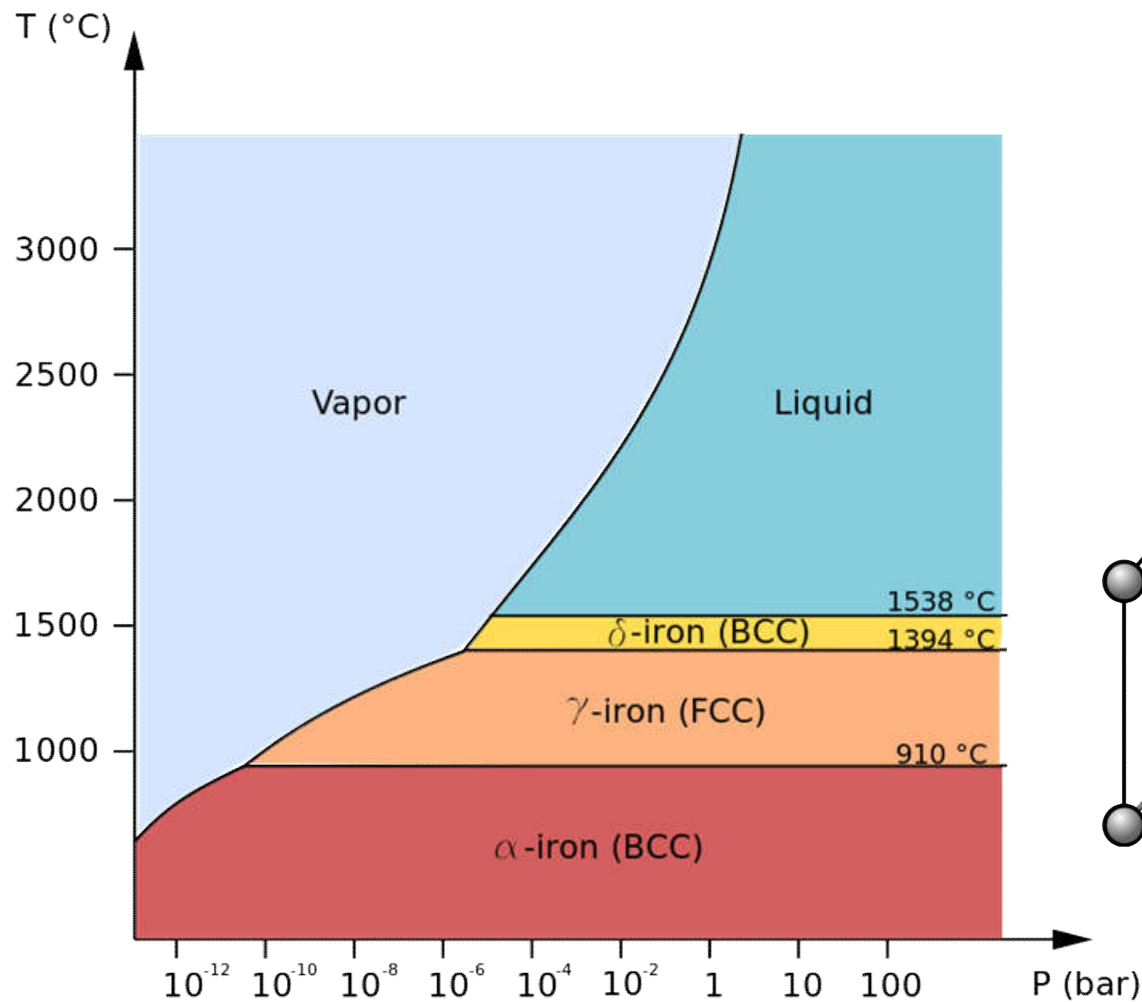
Allotrope 同素异构体

- ZnO can have Zinc Blende or Wurtzite structures

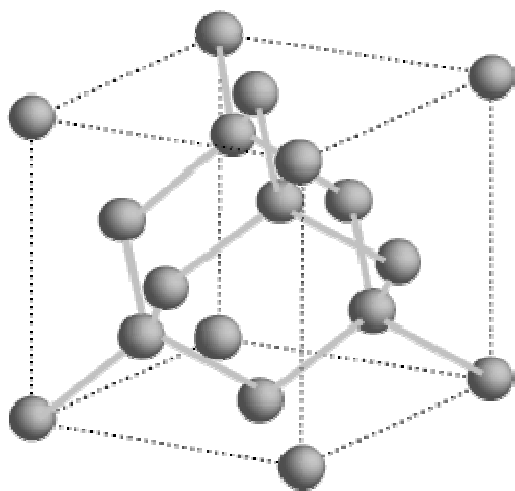


Allotrope 同素异构体

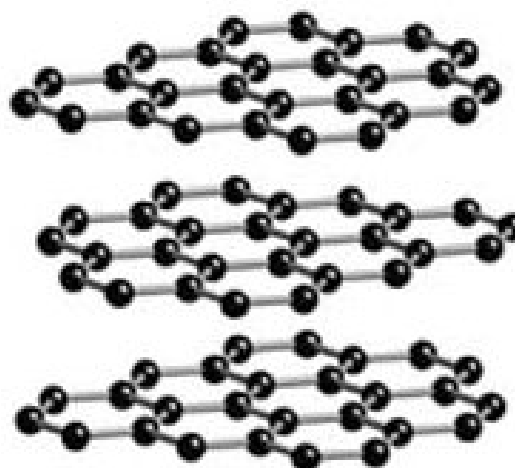
- Fe can have BCC or FCC structures



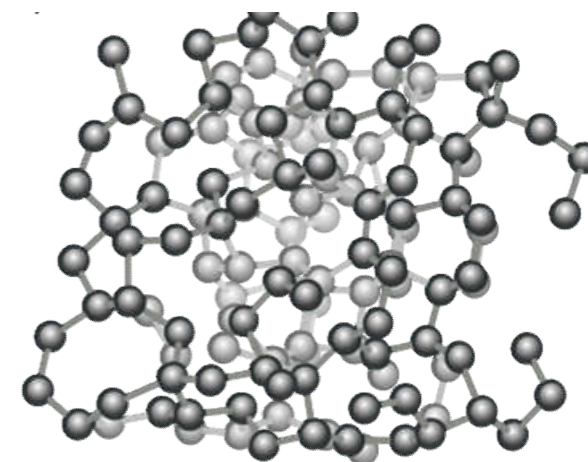
Allotropes for Carbon



diamond



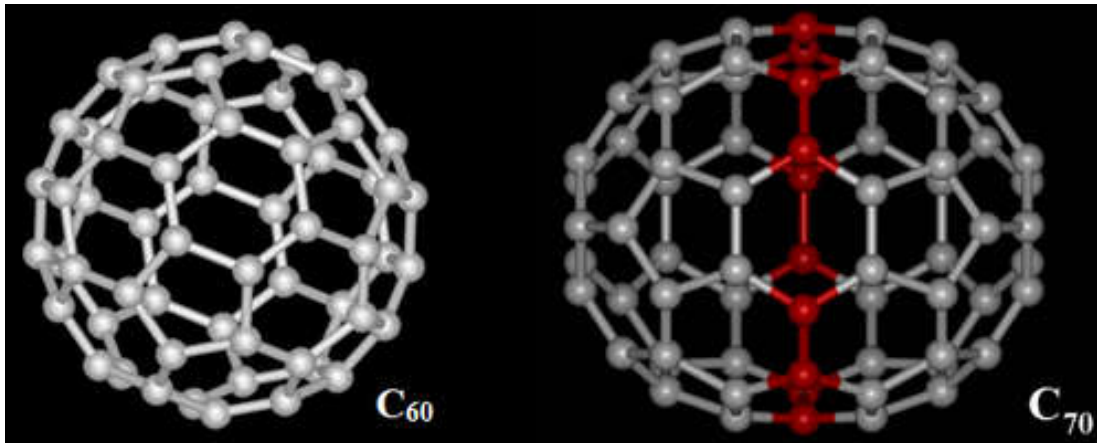
graphite



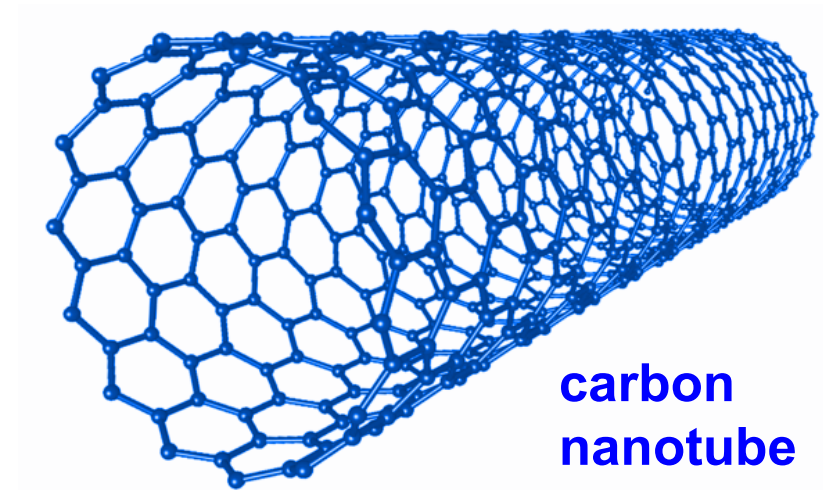
**amorphous
carbon**

Q: which one is electrically conductive, diamond or graphite?

Allotropes for Carbon

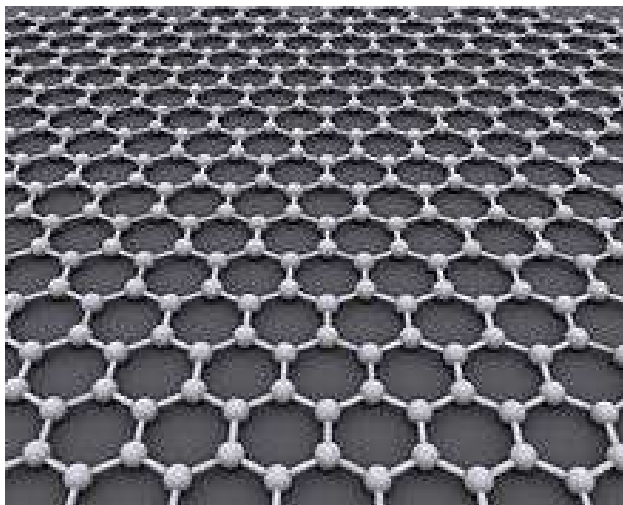


H. Kroto, R. Curl, R. Smalley
1996 Nobel Prize in Chemistry



**carbon
nanotube**

S. Iijima, *Nature* **354**, 56 (1991)

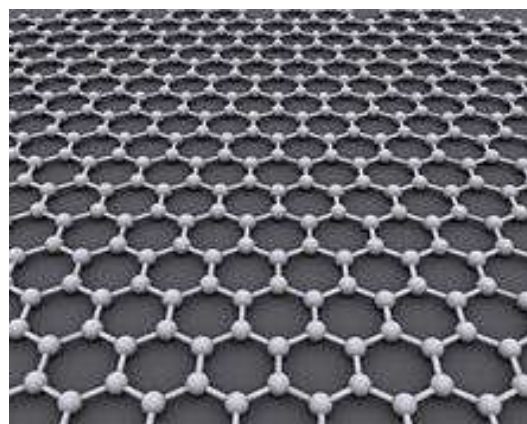


graphene

A. Geim, K. Novoselov
2010 Nobel Prize in Physics

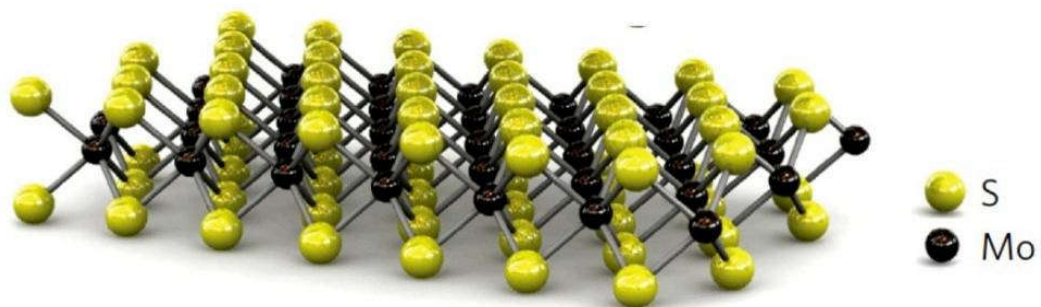
2D Materials

- Single atomic layer crystal

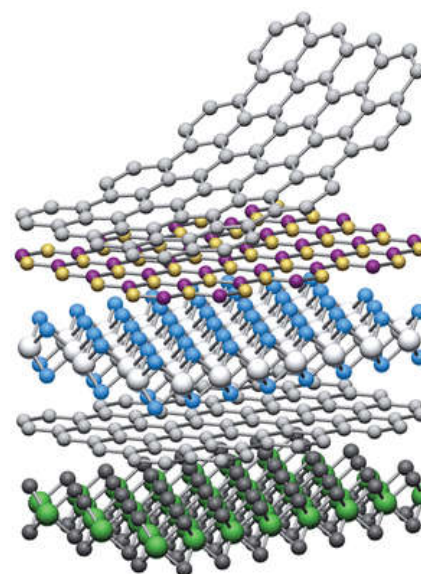


graphene

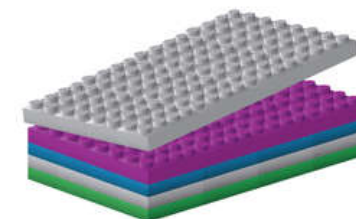
A. Geim, K. Novoselov
2010 Nobel Prize in Physics



Transition metal dichalcogenide (TMDC)
MoS₂, WSe₂, ...



	Graphene	
	hBN	
	MoS ₂	
	WSe ₂	
	Fluorographene	



Materials are Imperfect

"Crystals are like people, it is the defects in them which tend to make them interesting. "

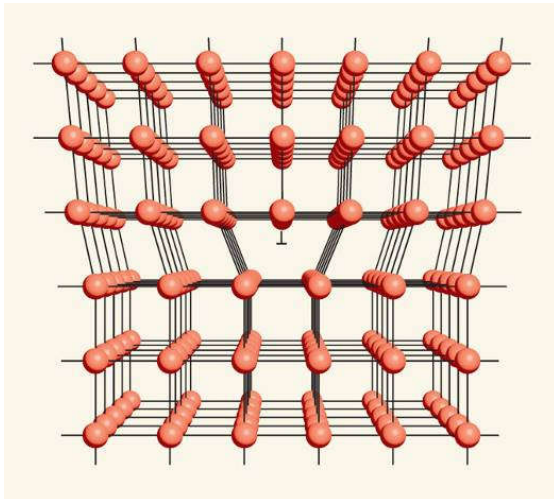
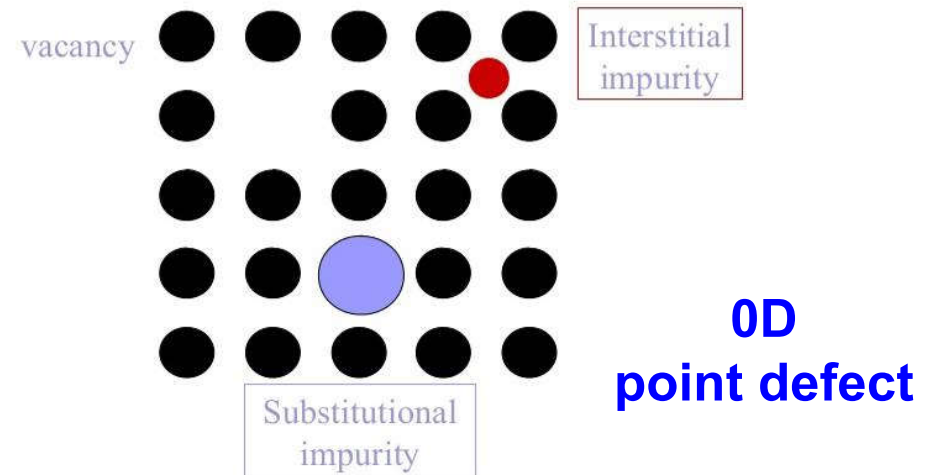
---- Colin Humphreys



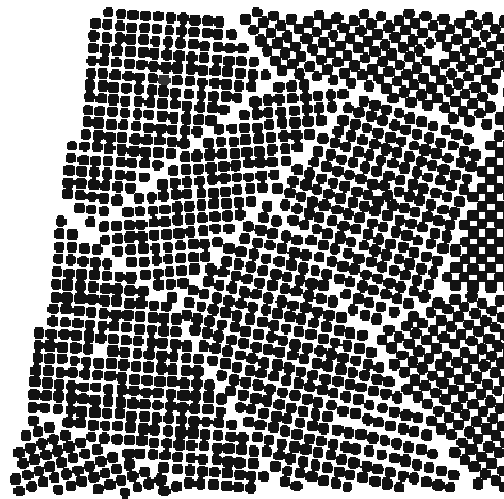
- **Defects**
 - 0D, 1D, 2D, 3D
- **Crystallinity**
 - Single Crystal, Polycrystal, Amorphous
 - Quasi-Crystal
 - Liquid Crystal

Defects in Crystals 缺陷

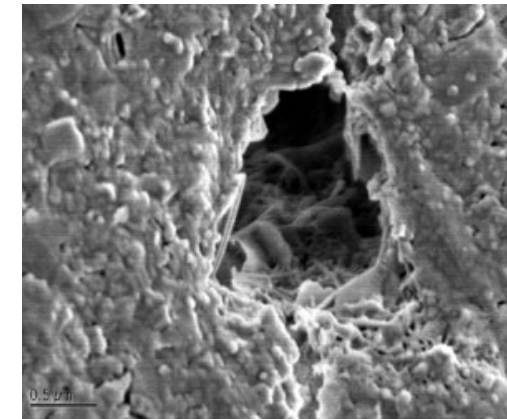
- 0D point defect 点缺陷
- 1D line defect 线缺陷
- 2D plane defect 面缺陷
- 3D volume defect 体缺陷



1D
dislocation

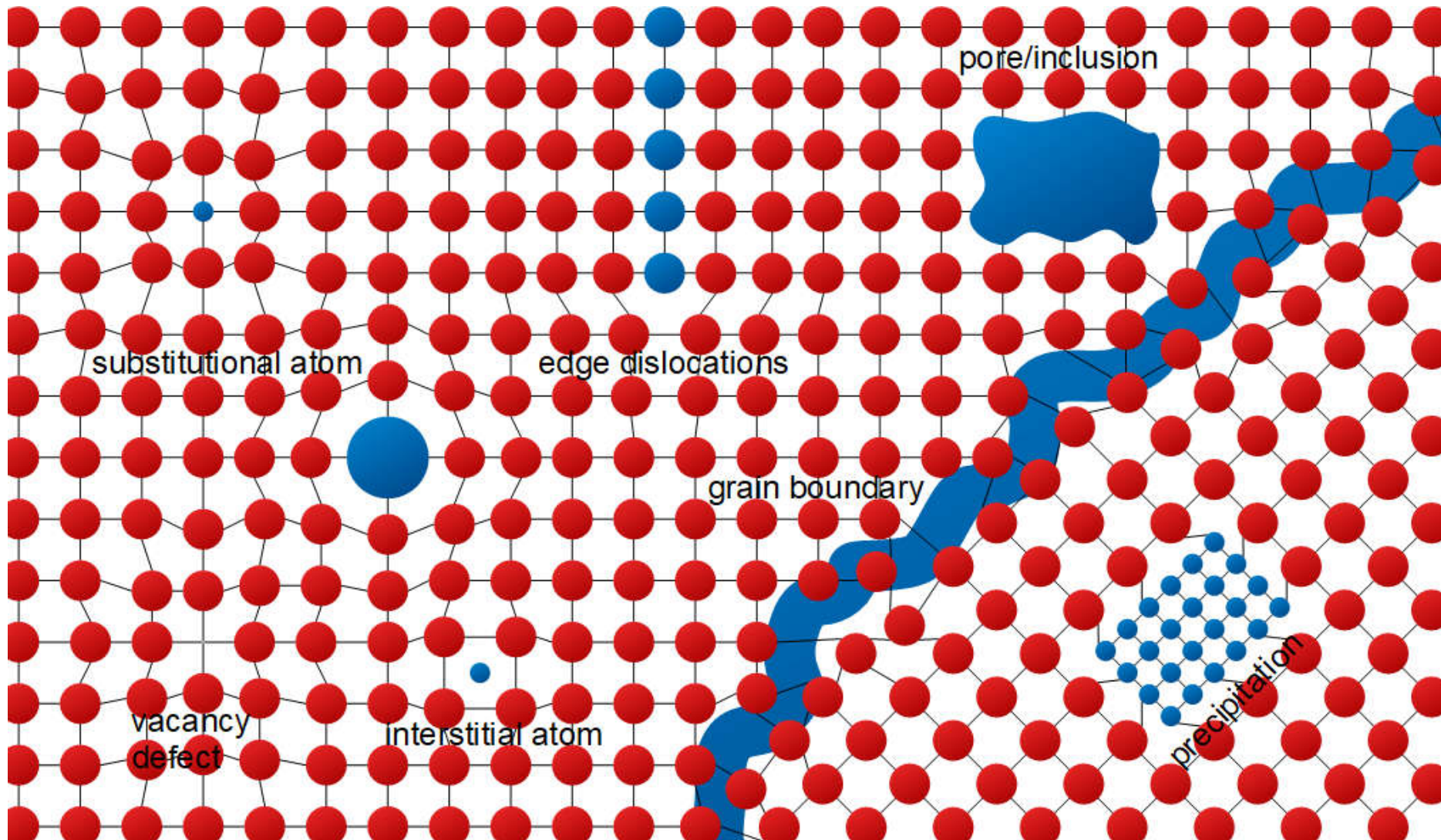


2D
grain boundary

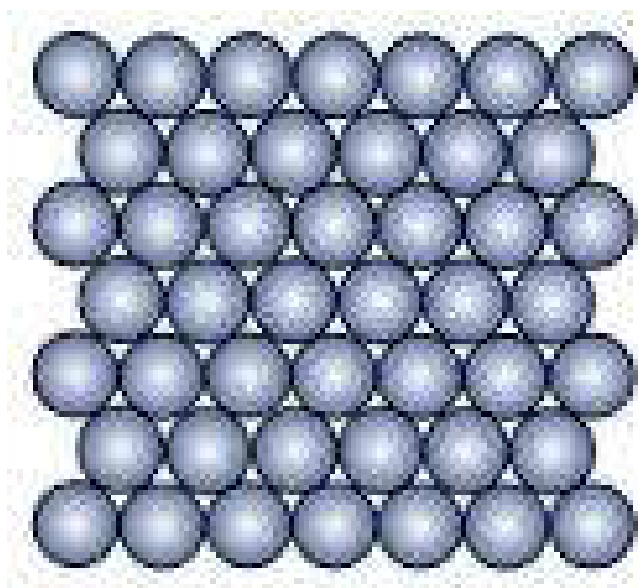


3D
void / precipitate

Defects in Crystals 缺陷

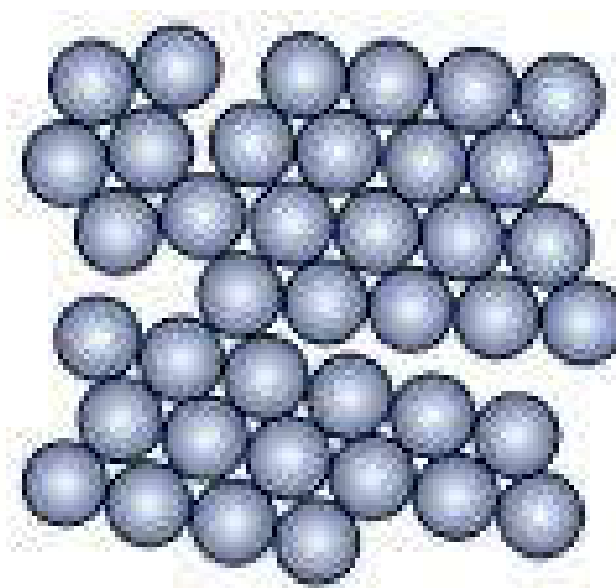


Crystallinity



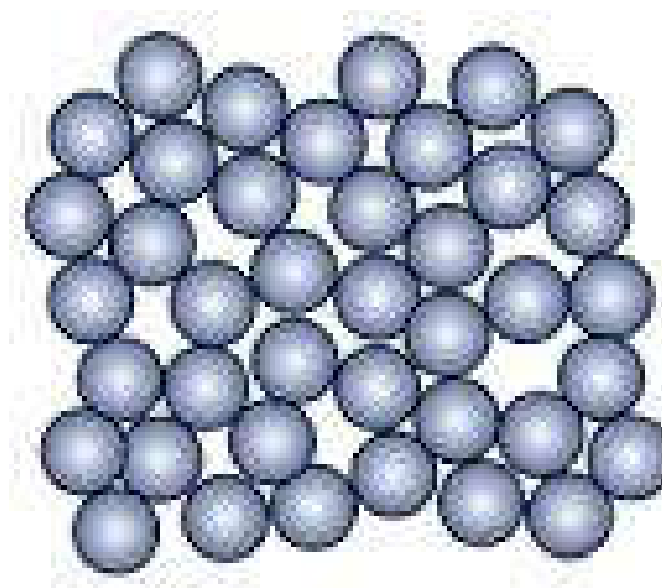
single crystalline
monocrystalline

单晶



polycrystalline
multicrystalline

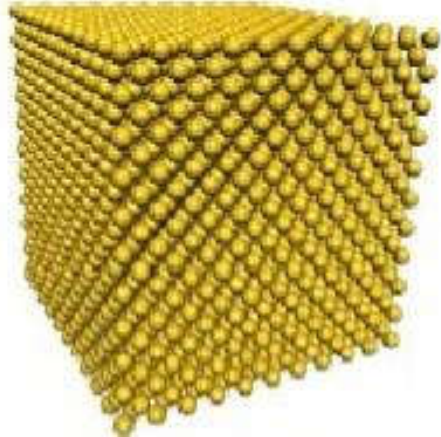
多晶



amorphous

非晶

Single Crystal / Mono Crystal



Quartz



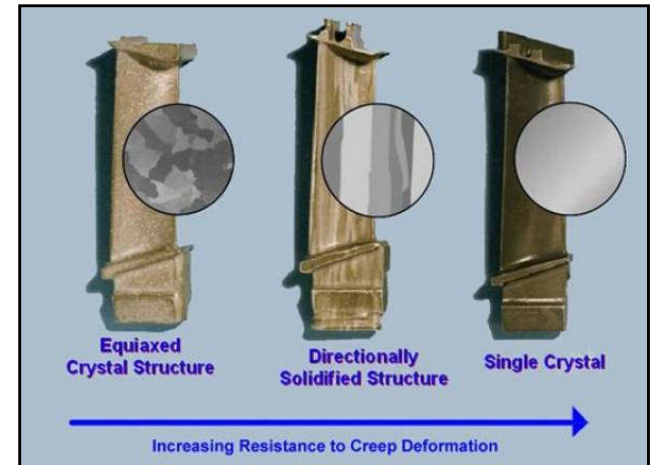
Sugar



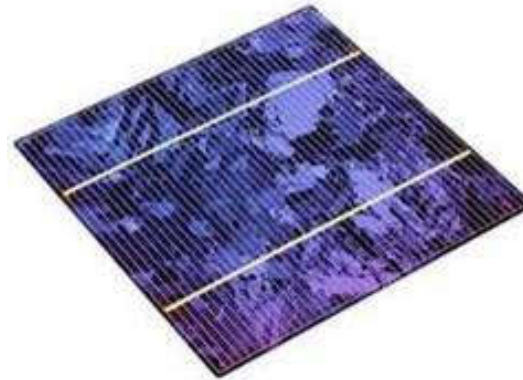
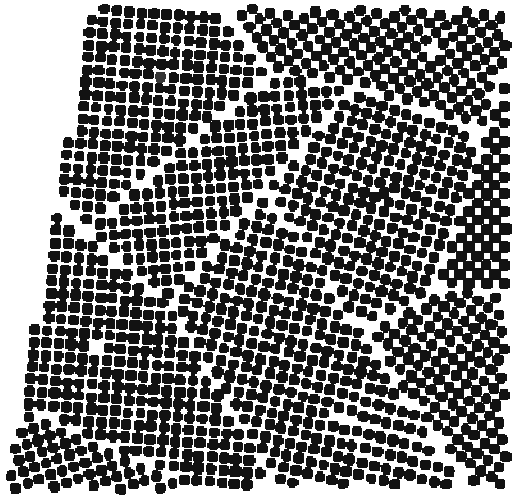
Silicon wafers,
GaAs, GaN, sapphire, ...



turbine blade



Polycrystal / Multicrystal

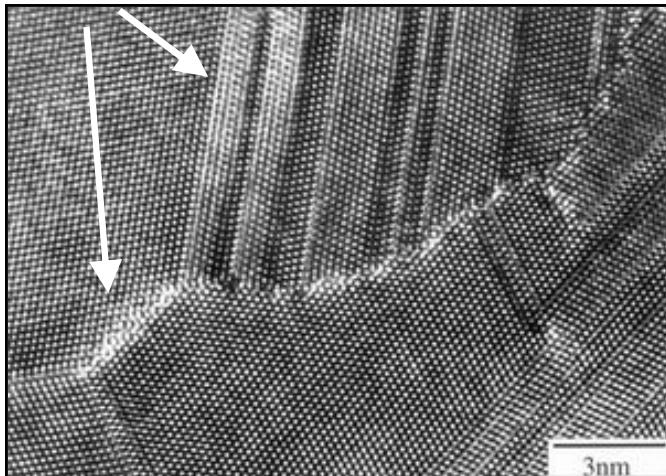


Poly-Crystalline
Solar Cell



Mono-Crystalline
Solar Cell

grain boundary



polycrystalline silicon



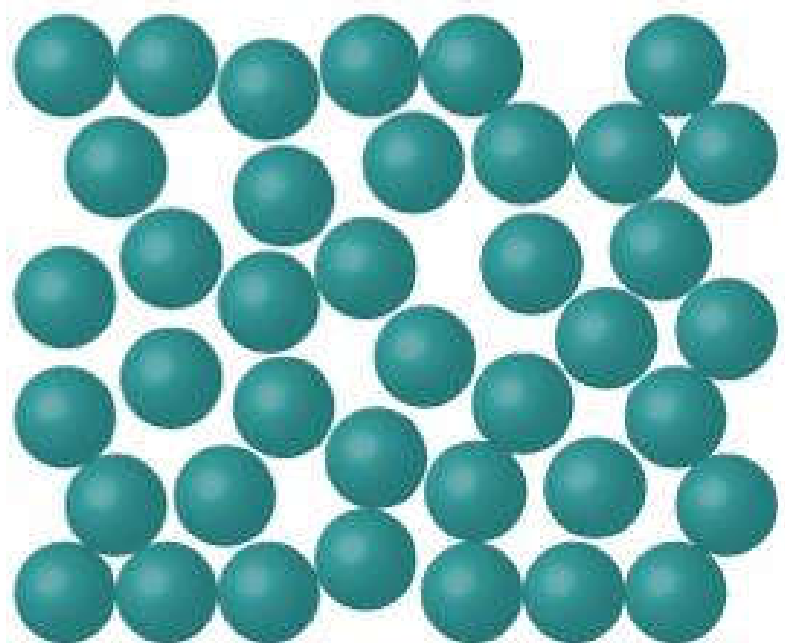
metals



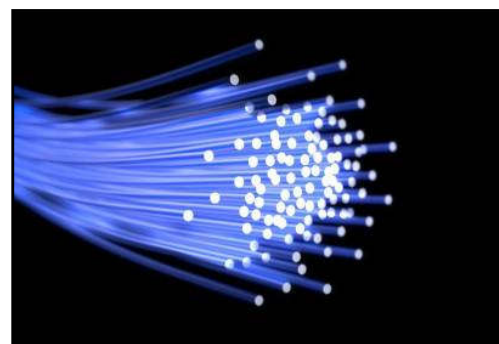
ceramics

Amorphous Materials

- Defects are everywhere ...



Amorphous



silica fiber



glass



plastics

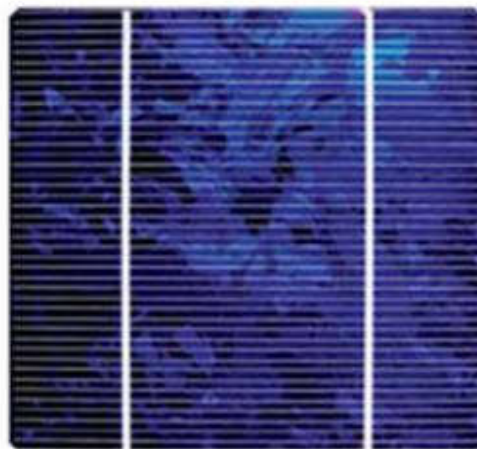
Crystallinity

Silicon Solar Cells

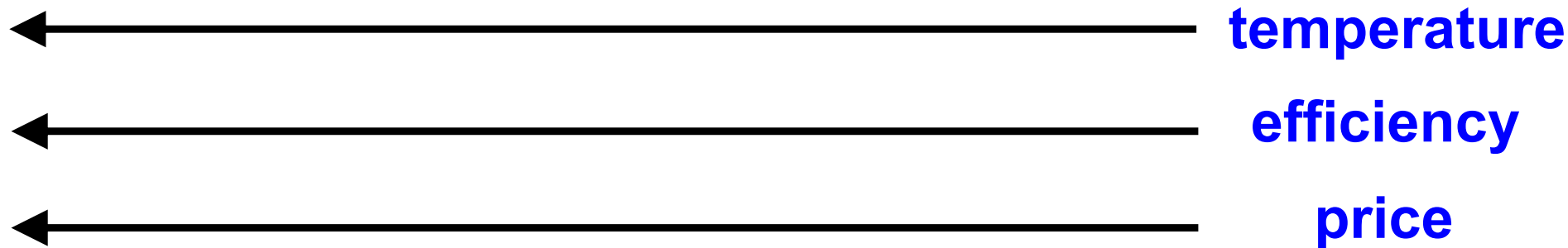
Monocrystalline



Polycrystalline

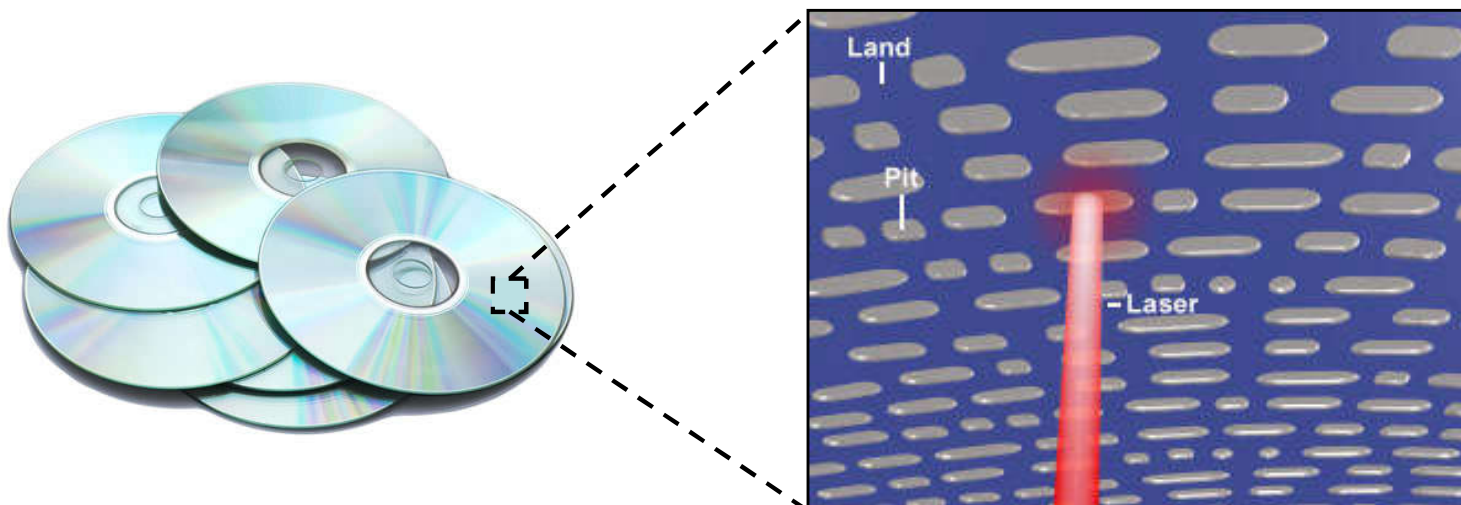


Amorphous

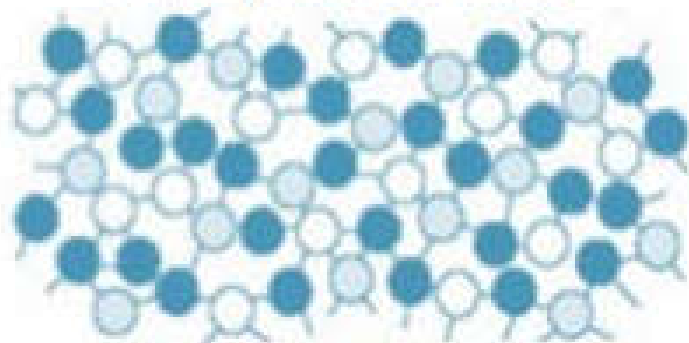


Optical Disc

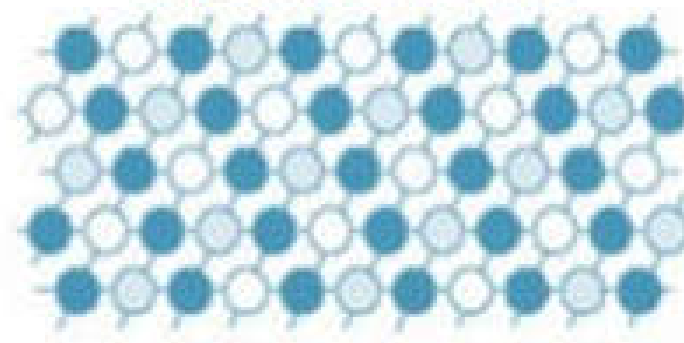
- Phase Change Memory



Amorphous Phase

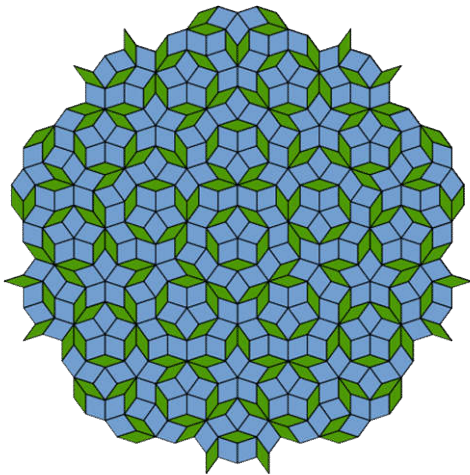


Crystalline Phase

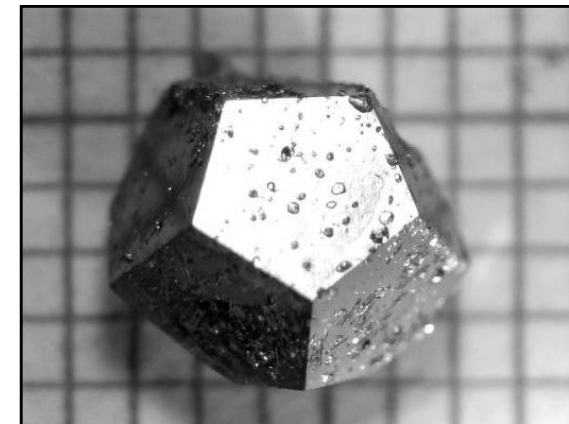
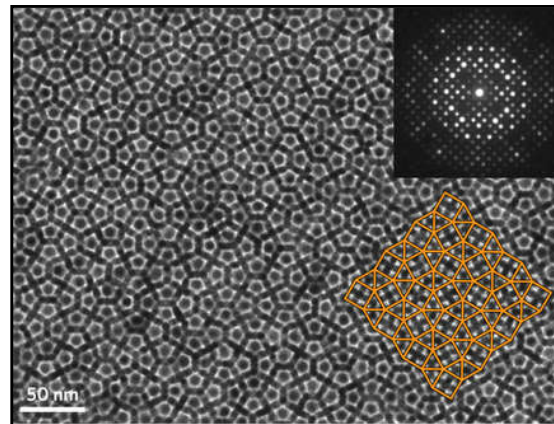


Quasi-Crystal 准晶

- **Neither crystalline nor amorphous**
 - **5, 8, 10, or 12-fold rotational symmetry**
 - **no translational symmetry**



Penrose tiling



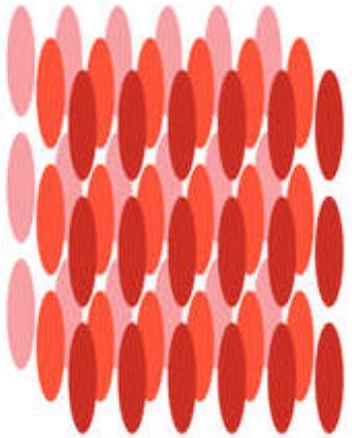
A Ho-Mg-Zn quasicrystal



D. Shechtman
2011 Nobel Prize in Chemistry

Liquid Crystals 液晶

Crystalline Solid



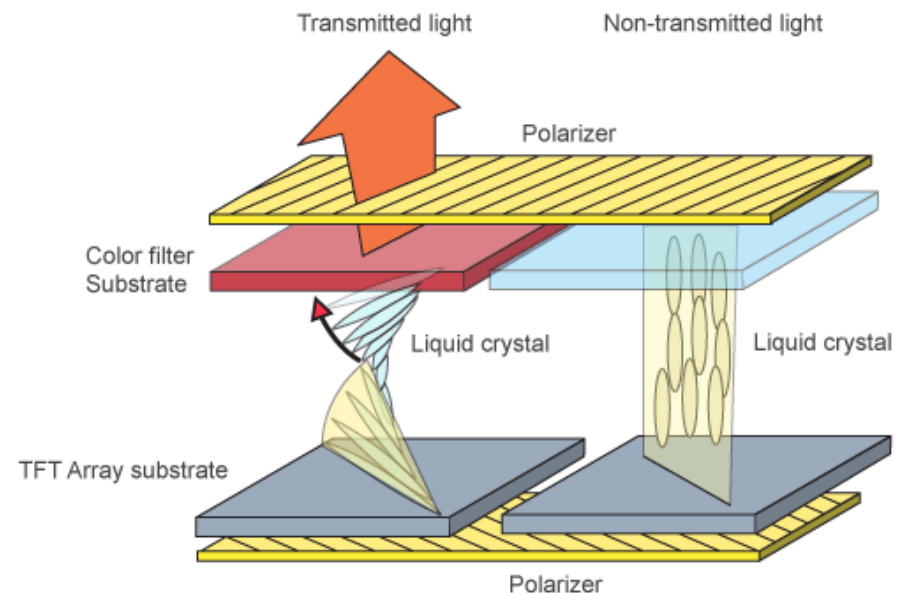
Liquid Crystal



Isotropic Liquid



Liquid crystal display (LCD)



P. de Gennes
1991 Nobel Prize in Physics

Diagram 2: The Fundamental Photonics of Liquid Crystal (Twisted Nematics)

Thank you for your attention